Phu Cuong Nguyen

Advanced Analysis for Three-Dimensional Semi-Rigid Steel Frames subjected to Static and Dynamic Loadings

Doctoral Thesis / Dissertation
YOUR KNOWLEDGE HAS VALUE

- We will publish your bachelor's and master's thesis, essays and papers

- Your own eBook and book - sold worldwide in all relevant shops

- Earn money with each sale

Upload your text at www.GRIN.com and publish for free
Advanced Analysis for Three-Dimensional Semi-Rigid Steel Frames subjected to Static and Dynamic Loadings
GRIN - Your knowledge has value

Since its foundation in 1998, GRIN has specialized in publishing academic texts by students, college teachers and other academics as e-book and printed book. The website www.grin.com is an ideal platform for presenting term papers, final papers, scientific essays, dissertations and specialist books.

Visit us on the internet:
http://www.grin.com/
http://www.facebook.com/grincom
http://www.twitter.com/grin_com
Advanced Analysis for Three-Dimensional Semi-Rigid Steel Frames subjected to Static and Dynamic Loadings

Nguyen Phu Cuong

The Graduate School
Sejong University
Department of Civil & Environmental Engineering
Advanced Analysis for Three-Dimensional Semi-Rigid Steel Frames subjected to Static and Dynamic Loadings

Nguyen Phu Cuong

A Dissertation Submitted to the Department of Civil & Environmental Engineering and the Graduate School of Sejong University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

June 2014

Approved by

Seung-Eock Kim

Major Advisor
This certifies that the dissertation of Nguyen Phu Cuong is approved.

The Graduate School

Sejong University

June 2014
DEDICATION

This dissertation is dedicated to my parents, my wife, and my daughters.
ACKNOWLEDGEMENTS

First of all, I would like to express my deep gratitude and respect to my advisor, Professor Seung-Eock Kim, for his guidance and support throughout my doctoral course in South Korea. The professional knowledge and research methodology that I learned from him have been useful in order to pursue my goal in life. In addition, I wish to thank Professors Seung-Eock Kim, Hyuk-Chun Noh, Jong-Jae Lee, Dong-Joo Kim, Jae-Hong Lee, Nam-Shik Ahn and Ki-Hak Lee for their valuable lectures and warm-hearted treatment towards me over the years.

Besides my adviser, I wish to thank my thesis committee members for their guidance and positive comments on my thesis. I greatly appreciate Professor Nguyen-Vu Duong, Director, John Von Neumann Institute - Vietnam National University HCMC, who firstly taught me scientific research methodology.

I am grateful for the assistance from all my friends in the Vietnamese Student Association and in the Bridge and Steel Structure Laboratory at Sejong University. I can never forget about what you have shared with me in happiness. I wish you all brilliant success in your future.

The financial support given by both Sejong University and the Bridge and Steel Structure Laboratory for my doctoral course is greatly appreciated.

Finally, the great motivation that helped me to overcome obstacles during the course of this research was the endless encouragement and support from my family, especially my wife and my parents. I would like to thank deeply them for this invaluable sacrifice and love. I would like to dedicate this dissertation to my parents Van-Phu and My-Le, my lovely daughters Dan-Chau and Lam-Chau and my wife Ngoc-Chi.

Seoul, June 2014

Nguyen Phu Cuong
# TABLE OF CONTENTS

**ADVANCED ANALYSIS FOR THREE-DIMENSIONAL SEMI-RIGID STEEL FRAMES SUBJECTED TO STATIC AND DYNAMIC LOADINGS**

**NGUYEN PHU CUONG**

**DEDICATION**

**ACKNOWLEDGEMENTS**

**TABLE OF CONTENTS** ................................................................. I

**LIST OF FIGURES** ........................................................................ V

**LIST OF TABLES** ........................................................................... XII

**ABSTRACT** .................................................................................. XIV

**CHAPTER 1. INTRODUCTION** .......................................................... 1

1. Motivation .................................................................................. 1
2. Objective and Scope .................................................................... 3
3. Organization of Dissertation .................................................... 5

*THIS DISSERTATION IS SUMMARIZED FROM JOURNAL ARTICLES* .......................................... 6

**CHAPTER 2. SECOND-ORDER SPREAD-OF-PLASTICITY APPROACH FOR NONLINEAR DYNAMIC ANALYSIS OF TWO-DIMENSIONAL SEMI-RIGID STEEL FRAMES** ................................. 7

1. Introduction ................................................................................ 7
2. Nonlinear Finite Element Formulation ........................................ 10
   2.1 Second-Order Spread-of-Plasticity Beam-Column Element .......... 10
   2.2 Nonlinear Beam-to-Column Connections .................................. 23
      2.2.1 Modified Tangent Stiffness Matrix including Nonlinear Connections 23
      2.2.2 Moment-Rotation Relationship of Nonlinear Connections .......... 28
      2.2.3 Cyclic Behavior of Nonlinear Connections ............................. 29
3. Nonlinear Solution Procedures .................................................... 31
CHAPTER 3. SECOND-ORDER PLASTIC-HINGE APPROACH FOR NONLINEAR STATIC AND DYNAMIC ANALYSIS OF THREE-DIMENSIONAL SEMI-RIGID STEEL FRAMES..... 62

1. Introduction ........................................................................................................... 62
2. Nonlinear Finite Element Formulation ................................................................ 65
   2.1 Second-Order Plastic-Hinge Beam-Column Element ..................................... 65
      2.1.1 Stability Functions accounting for Second-Order Effects .................... 65
      2.1.2 Refined Plastic Hinge Model accounting for inelastic effects .......... 68
      2.1.3 Shear Deformation Effect ................................................................... 71
      2.1.4 Element Stiffness Matrix accounting for $P−\Delta$ Effect .................. 73
   2.2 Semi-rigid Connection Element ................................................................... 77
      2.2.1 Element Modeling ................................................................................ 77
      2.2.2 Semi-Rigid Connection Models for Rotational Springs ..................... 78
      2.2.3 Cyclic Behavior of Rotational Springs ............................................ 80
3. Nonlinear Solution Procedures ......................................................................... 81
   3.1 Nonlinear Static Algorithm .......................................................................... 81
      3.1.1 Formulation ....................................................................................... 81
      3.1.2 Application ....................................................................................... 85
   3.2 Nonlinear Dynamic Algorithm .................................................................... 89
      3.2.1 Formulation ....................................................................................... 89
      3.2.2 Application ....................................................................................... 92
4. Numerical Examples and Discussions ................................................................ 94
   4.1 Static Problems ............................................................................................. 94
      4.1.1 Vogel 2-D Portal Steel Frame ............................................................. 94
      4.1.2 Stelmack Experimental 2-D Two-Story Steel Frame ......................... 95
CHAPTER 4.  
SECOND-ORDER SPREAD-OF-PLASTICITY APPROACH FOR NONLINEAR 
STATIC AND DYNAMIC ANALYSIS OF THREE-DIMENSIONAL SEMI-RIGID STEEL 
FRAMES  .............................................................................................................. 139

1. Introduction......................................................................................................... 139
2. Nonlinear Finite Element Formulation ............................................................... 142
   2.1 Second-Order Spread-of-Plasticity Beam-Column Element....................... 142
      2.1.1 The Effects of Small P-delta and Shear Deformation ......................... 142
      2.1.2 The Effect of Spread-of-Plasticity .................................................. 146
      2.1.3 Element Stiffness Matrix accounting for the Effect of Large P-delta 149
   2.2 Nonlinear Beam-to-Column Connection Element ................................. 151
      2.2.1 Element Modeling ........................................................................ 151
      2.2.2 Cyclic Behavior of Rotational Springs ....................................... 155
3. Nonlinear Solution Procedures .......................................................................... 156
   3.1 Nonlinear Static Algorithm ........................................................................ 156
   3.2 Nonlinear Dynamic Algorithm .................................................................. 159
4. Numerical Examples and Discussions .............................................................. 164
   4.1 Static Problems .......................................................................................... 164
      4.1.1 Vogel Portal Steel Frame .............................................................. 164
      4.1.2 Stelmack Experimental Two-Story Steel Frame ......................... 167
      4.1.3 Vogel Six-Story Steel Frame ....................................................... 169
      4.1.4 Orbison Six-Story Space Steel Frame – A Case Study ............ 174
4.2 Dynamic Problems .......................................................................................... 177
  4.2.1 Portal Steel Frame subjected to Earthquakes.............................................. 178
  4.2.2 Experimental Two-Story Steel Frame subjected to Cyclic Loadings. 184
  4.2.3 Space Six-Story Steel Frame – A Case Study ........................................... 187
5. Summary and Conclusions ............................................................................... 195

CHAPTER 5. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS ............ 197
  1. Summary and Conclusions ............................................................................ 197
  2. Recommendations.......................................................................................... 200

REFERENCES ......................................................................................................... 203

국문초록 ............................................................................................................. 209
LIST OF FIGURES

Fig. 2.1 Beam-column element modeling under arbitrary loads ........................................ 10

Fig. 2.2 Meshing of beam-column element into n sub-elements.................................... 11

Fig. 2.3 Illustration of meshing of element cross-section and states of fibers .............. 12

Fig. 2.4 Constitutive model is assumed for steel material ............................................ 13

Fig. 2.5 Beam-column member including nonlinear connections with eight degrees of freedom ........................................................................................................................................ 23

Fig. 2.6 Modified beam-column member with conventional six degrees of freedom .... 24

Fig. 2.7 The independent hardening model .................................................................. 31

Fig. 2.8 Earthquake records ....................................................................................... 37

Fig. 2.9 Portal frame subjected to earthquakes .......................................................... 38

Fig. 2.10 Displacement time-history responses of portal frame under Loma Prieta earthquake ................................................................. 41

Fig. 2.11 Displacement time-history responses of portal frame under San Fernando earthquake ................................................................. 42

Fig. 2.12 Second-order inelastic responses of portal frame with and without residual stress .............................................................................................................. 43

Fig. 2.13 Cut section A-A and fiber no. 1 is being monitored ..................................... 43

Fig. 2.14 Plastic deformation and stress at fiber no. 1 in cut section A-A in portal frame with and without residual stress ......................................................... 45

Fig. 2.15 Plastic deformation and stress at fiber no. 1 in cut section A-A in portal frame with and without residual stress ......................................................... 46

Fig. 2.16 Two-story steel frame with nonlinear connections ....................................... 47
Fig. 2.17. Second-order elastic responses of two-story frame for various connection
........................................................................................................................................ 49
Fig. 2.18. Second-order inelastic responses of two-story frame for various connections
........................................................................................................................................ 50
Fig. 2.19. Hysteresis loops at connection C of two-story frame ................................. 51
Fig. 2.20. Vogel six-story steel frame with semi-rigid connections............................ 52
Fig. 2.21. Second-order elastic displacement responses at roof floor of Vogel frame
under forced loadings – without bowing effects ............................................................ 55
Fig. 2.22. Second-order elastic displacement responses at roof floor of Vogel frame
under forced loadings – with bowing effects................................................................. 56
Fig. 2.23. Moment-rotation responses at connection C of Vogel frame under forced
loadings with $\omega = 1.66 \text{ rad/sec}$ in the second-order elastic analysis ....................... 56
Fig. 2.24. Second-order inelastic displacement responses at roof floor of Vogel frame
under El Centro earthquake considering geometric imperfections .......................... 58
Fig. 2.25. Moment-rotation responses at connection C of Vogel frame under El Centro
earthquake considering geometric imperfections in the second-order inelastic analysis...
........................................................................................................................................ 58
Fig. 3.1 Full plastification surfaces ........................................................................... 70
Fig. 3.2 Element end force and displacement notations .......................................... 74
Fig. 3.3 Space connection element model with zero-length .................................... 77
Fig. 3.4. The independent hardening model ............................................................... 80
Fig. 3.5 General characteristic of nonlinear systems ............................................... 82
Fig. 3.6 Characteristics of $GSP$ ............................................................................... 85
Fig. 3.7 Flow chart of the GDC algorithm ................................................................. 88
Fig. 3.8 Flow chart of the proposed algorithm for dynamic analysis ......................... 94
Fig. 3.9. Vogel portal frame with semi-rigid connections ........................................... 95
Fig. 3.10. Load – displacement response of Vogel portal frame (PZ: plastic-zone
method, PH: plastic-hinge method) ............................................................................. 96
Fig. 3.11. Stelmack experimental two-story frame ......................................................... 97
Fig. 3.12. Moment – rotation behavior of Stelmack two-story frame ............................. 97
Fig. 3.13. Load – displacement response of Stelmack two-story frame ........................ 98
Fig. 3.14. Liew SRF3 portal frame ............................................................................. 99
Fig. 3.15. Moment – rotation relations of SRF3 portal frame .................................... 100
Fig. 3.16. Load – displacement response of Liew SRF3 portal frame ......................... 100
Fig. 3.17. Orbison six-story space frame with semi-rigid connections ......................... 101
Fig. 3.18. Load – displacement response at Y direction of Orbison six-story frame ... 102
Fig. 3.19. Chan 2-D two-story steel frame ................................................................... 104
Fig. 3.20. Second-order elastic responses of 2-D two-story frame ............................ 107
Fig. 3.21. Second-order inelastic responses of 2-D two-story frame ........................... 109
Fig. 3.22. Hysteresis loops at the connection C for various analyses of 2-D two-story
frame ............................................................................................................................ 110
Fig. 3.23. Geometry and loads of Vogel six-story frame ............................................. 111
Fig. 3.24. Time-displacement response by second-order elastic analysis (\(\omega = 1.00 \text{ rad/s}\))
......................................................................................................................................... 114
Fig. 3.25. Time-displacement response by second-order elastic analysis (\(\omega = 1.66 \text{ rad/s}\))
......................................................................................................................................... 114
Fig. 3.26. Time-displacement response by second-order elastic analysis ($\omega = 2.41$ rad/s) ...................................................................................................................................... 115

Fig. 3.27. Time-displacement response by second-order elastic analysis ($\omega = 3.30$ rad/s) ...................................................................................................................................... 115

Fig. 3.28. Time-displacement response by second-order elastic analysis under sudden load during 1s: $F_1(t) = 10.23kN$, $F_2(t) = 20.44kN$ ....................................................... 116

Fig. 3.29. Comparing time-displacement response of the three models ($\omega = 1.66$ rad/s) ...................................................................................................................................... 116

Fig. 3.30. Comparing hysteresis loops of the three models at connection C ($\omega = 1.66$ rad/s) ...................................................................................................................................... 117

Fig. 3.31. Dimensions and properties of Chan space two-story frame......................... 118

Fig. 3.32. Time-displacement response at node A in nonlinear elastic analysis .......... 119

Fig. 3.33. Moment-rotation curve of nonlinear strong-axis spring at connection C .... 120

Fig. 3.34. Moment-rotation curve of nonlinear weak-axis spring at connection C...... 120

Fig. 3.35. 3-D two-story frame subjected to earthquakes............................................. 121

Fig. 3.36. Nonlinear time-history responses of 3-D two-story frame under El Centro earthquake..................................................................................................................... 125

Fig. 3.37. Nonlinear time-history responses of 3-D two-story frame under Northridge earthquake..................................................................................................................... 127

Fig. 3.38. Nonlinear time-history responses of 3-D six-story steel frame under Northridge earthquake ..................................................................................................................... 132

Fig. 3.39. Nonlinear time-history responses of 3-D six-story steel frame under San Fernando earthquake..................................................................................................................... 134
Fig. 3.40. Hysteresis loops of strong-axis rotational spring at connection C for various analyses of 3-D six-story steel frame subjected to El Centro earthquake .................... 135
Fig. 3.41. Hysteresis loops of strong-axis rotational spring at connection C for various analyses of 3-D six-story steel frame subjected to San Fernando earthquake............ 136
Fig. 4.1 Discretization of beam-column element ......................................................... 145
Fig. 4.2 The ECCS residual stress pattern for I-section ............................................. 147
Fig. 4.3 Modeling of space connection element with zero-length........................... 152
Fig. 4.4. The independent hardening model .............................................................. 155
Fig. 4.5. Vogel portal frame with semi-rigid connections......................................... 164
Fig. 4.6. Load – displacement curve of Vogel portal rigid frame with and without shear deformation.............................................................................................................. 166
Fig. 4.7. Load – displacement curve of Vogel portal semi-rigid frame ................. 166
Fig. 4.8. Stelmack two-story frame ......................................................................... 167
Fig. 4.9. Moment – rotation curve of connections by Stelmack experiment and curve fitting .................................................................................................................... 168
Fig. 4.10. Load – displacement curve of Stelmack two-story frame ...................... 169
Fig. 4.11. Vogel six-story frame with semi-rigid connections................................. 170
Fig. 4.12. Load – displacement curve of Vogel six-story rigid frame (PZ – Plastic Zone, RPH – Refined Plastic Hinge) ............................................................................. 171
Fig. 4.13. Moment-rotation curve of semi-rigid connections for Vogel six-story frame ......................................................................................................................... 173
Fig. 4.14. Load – displacement curve of Vogel six-story frame with different beam-to-column connections ......................................................................................... 173
Fig. 4.15. Orbison six-story space frame with semi-rigid connections ................. 174

Fig. 4.16. Load – displacement curve at Y-direction node A of Orbison 3-D six-story frames ........................................................................................................................................................................... 176

Fig. 4.17. Load – displacement curve at Y-direction node A of Orbison 3-D six-story frames with and without initial member out-of-straightness........................................ 177

Fig. 4.18. Portal frame subjected to earthquakes.................................................. 178

Fig. 4.19. Displacement time-history responses of portal frame under Loma Prieta earthquake..................................................................................................................... 181

Fig. 4.20. Displacement time-history responses of portal frame under San Fernando earthquake..................................................................................................................... 182

Fig. 4.21. Second-order inelastic responses of portal frame with and without residual stress ............................................................................................................................. 183

Fig. 4.22. Experimental two-story steel frame ...................................................... 185

Fig. 4.23. Comparison of moment-rotation curves at connection C of Stelmack experimental frame..................................................................................................................... 187

Fig. 4.24. Nonlinear time-history responses of six-story space steel frame subjected to El Centro earthquake ........................................................................................................ 190

Fig. 4.25. Nonlinear time-history responses of six-story space steel frame subjected to San Fernando earthquake..................................................................................................................... 191

Fig. 4.26. Second-order inelastic displacement responses of six-story space steel frame under two earthquakes considering geometric imperfections ................................. 193

Fig. 4.27. Moment-rotation responses at connection C of six-story space steel frame under two earthquakes considering geometric imperfections ................................. 194
LIST OF TABLES

Table 2.1 Peak ground acceleration and its corresponding time steps of earthquake records (PEER, 2011) .................................................................................................................... 35

Table 2.2 Comparison of first two natural periods (s) along the applied earthquake direction of portal frame ........................................................................................................ 39

Table 2.3 Comparison of peak displacements (mm) of portal frame .................................. 39

Table 2.4 Periods and Rayleigh damping coefficients of Vogel frame ................................. 53

Table 3.1 Curve-fitting connection parameters of Liew SRF3 portal frame ....................... 99

Table 3.2 Peak displacements (mm) of 2-D two-story steel frame .................................. 106

Table 3.3 Comparison of fundamental natural frequencies $\omega$ (rad/s) .............................. 112

Table 3.4 Comparison of fundamental natural frequencies $\omega$ (rad/s) .............................. 122

Table 3.5 Parameters of semi-rigid connections follow the Kishi-Chen model ................. 129

Table 3.6 Comparison of first two natural periods (s) along applied earthquake direction of 3-D six-story steel frame ...................................................................................... 129

Table 3.7 Comparison of peak displacements (mm) at node A of 3-D six-story steel frame .................................................................................................................................. 130

Table 4.1 Comparison of ultimate load factor of Vogel portal frame .............................. 165

Table 4.2 Comparison of ultimate load factor of Vogel six-story rigid frame .................. 172

Table 4.3 Parameters of connections for the Chen-Lui exponential model ...................... 172

Table 4.4 Parameters of semi-rigid connections follow the Kishi-Chen model ............... 176

Table 4.5 Comparison of periods and Rayleigh damping coefficients of portal frame .................................................................................................................... 179

Table 4.6 Comparison of peak displacements (mm) of portal frame ............................... 180
Table 4.7 Comparison of peak displacements (mm) at node A of six-story space steel frame.................................................................................................................................................. 192
ABSTRACT

Advanced Analysis for Three-Dimensional Semi-Rigid Steel Frames subjected to Static and Dynamic Loadings

Nguyen Phu Cuong
Dept. of Civil & Environmental Engineering
The Graduate School
Sejong University

This dissertation presents three various advanced analysis approaches which can capture accurately and efficiently the ultimate strength and behavior of steel framed structures with nonlinear beam-to-column connections subjected to static and dynamic loadings. Three major sources of nonlinearity are considered in the analyses as follows: (1) material nonlinearity; (2) geometric nonlinearity; and (3) connection nonlinearity. Three types of nonlinear beam-column element formulation considering both geometric and material nonlinearities are coded into two nonlinear structural analysis programs: (1) Nonlinear Structural Analysis Program (NSAP) – 2-D plastic-zone finite element; (2) Practical Advanced Analysis Program (PAAP) – 3-D refined plastic-hinge element and 3-D plastic-fiber element. Three types of steel frames analyzed by the proposed program are: (1) rigid frames – beam-to-column connections are fully rigid; (2) linear semi-rigid frames – beam-to-column connections have constant stiffness; and (3)
nonlinear semi-rigid frames – beam-to-column connections have continuously variable stiffness. Three types of analysis can be performed are: (1) nonlinear inelastic static analysis; (2) nonlinear elastic and inelastic time-history analysis; and (3) free vibration analysis. Three main resources of damping are taken into account in the proposed programs are: (1) hysteretic damping due to inelastic material; (2) structural viscous damping employing Rayleigh damping; (3) hysteretic damping due to nonlinear beam-to-column connections.

In the first approach – using the proposed program NSAP, a displacement-based finite element procedure for second-order spread-of-plasticity analysis of planar steel frames with nonlinear beam-to-column connections under dynamic and seismic loadings is developed. A partially strain-hardening elastic-plastic beam-column element, which directly takes into account geometric nonlinearity, bowing effects, gradual yielding of material, and flexibility of nonlinear connections, is proposed. The geometric nonlinearities are captured by using linear and Hermite interpolation functions through members are divided into several sub-elements. The spread of plasticity is considered by tracing the uniaxial stress-strain relationship of each fiber on the cross section of sub-elements. Nonlinear connections are simulated by zero-length rotational springs, which are taken into account by modifying the tangent stiffness matrix of the nonlinear beam-column element. A numerical procedure based on the combination of the Newmark numerical integration method and the Newton-Raphson equilibrium iterative algorithm is proposed for solving differential equations of motion. The nonlinear dynamic behavior predicted by the proposed program compare well with those given by the commercial finite element software ABAQUS and previous studies.
In the second and the third approaches – using the proposed program PAAP, for practical advanced analysis, the geometric nonlinearity caused by the interaction between axial force and bending moment is taken into account by using the stability functions, while the material nonlinearity is captured by using the refined plastic hinge model (the Refined Plastic-Hinge element - RPH) or using the spread-of-plasticity model (the Plastic-Fiber element - PF). The benefit of employing the stability functions and the refined plastic hinge model is that it can acceptably accurately capture the nonlinear effects by modeling one or two element per member, and hence this leads to a high computational efficiency compared to the finite element method using the interpolation functions. In some cases, it needs to obtain the higher level of accuracy, and it also gives more information for the member, the spread-of-plasticity model should be used. To capture accurately the spread-of-plasticity effect, the member is monitored through integration points along the member length and fibers on the cross-section. To consider the nonlinear behavior of semi-rigid connections, a spatial connection element with three translation springs and three rotation springs is developed to simulate the beam-to-column joint.

To solve nonlinear static equilibrium equations, the modified Newton-Raphson method and the generalized displacement control method is adopted herein because of their general numerical stability and efficiency. The generalized displacement control method can accurately trace the equilibrium path of nonlinear problems with multiple limit points and snap-back points. The modified Newton-Raphson method is utilized to apply fully static loads before the dynamic analysis procedure is executed. An incremental-iterative solution algorithm based on the Hilber-Hughes-Taylor method or
the Newmark direct integration combined with the Newton-Raphson equilibrium iterative method is adopted for solving equations of motion.

Two computer programs are developed: (1) Nonlinear Structural Analysis Program (NSAP) – written in the C++ programming language; (2) Practical Advanced Analysis Program (PAAP) – written in the FORTRAN 77 programming language. They are verified for accuracy and computational efficiency by comparing predicted results with those generated by the commercial finite element analysis packages of ABAQUS and SAP2000, and other results available in the literature. Through several numerical examples, the proposed program (PAAP) proves to be a reliable and efficient tool for daily practice design in lieu of using costly and time-consuming commercial software.
Chapter 1. INTRODUCTION

1. Motivation

It is generally recognized that steel framed structures exhibit significantly nonlinear behavior prior to achieving their ultimate load-carrying capacity or instability. Thus, a second-order inelastic analysis or an advanced analysis is the most exactly result for predicting the real performance of steel framed structures instead of using conventional analysis/design approach. Advanced analysis can efficiently capture the ultimate strength and stability of a whole structural system and its component members so that separate member capacity checks encompassed by specification equations are no longer necessary. While greater complexity is introduced in the analysis, a significant reduction in effort is achieved in the design assessment. This may be accomplished through an efficient final checking of both member and system limit states for a structure where preliminary member sizing was based on serviceability requirements. For steel framed structures, advanced analysis methods can be generally classified into two categories of plastic hinge and plastic zone approaches based on the level of refinement used to represent yielding.

The beam-column member in the plastic hinge approaches is modeled by an appropriate way to eliminate its further subdivision, and the plastic hinges representing the inelastic behavior of material are assumed to be lumped at both ends of the member. The refined plastic hinge method is one of plastic hinge approaches. In this method, the inelastic behavior in the member is modeled in terms of member forces instead of the detailed level of stress and strain as used in the plastic zone analysis, the yielding is
evaluated based on a yielding surface criteria. The principal advantages of this method are that it is simple in formulation as well as implementation and more importantly, it is relatively accurate for the assessment of strength and stability of a structural system by using the one or two elements per member in the modeling.

In the plastic zone approach, the beam-column member is discretized into several finite sub-elements along the member length, and the cross section of each sub-element is divided into several small fibers, of which the uniaxial stress-strain relationships of material are monitored during the analysis process. This method performs the spread of plasticity throughout the cross section and along the member length. Although the solution of this method is considered the “exact” solution and easily included the effects of local, flexural-torsional, and lateral-torsional buckling which are significant characteristics of steel structures, it is generally recognized to be too expensive in computational time and computer resources because a very refined discretization of the structure is necessary. Therefore, it is usually applicable only for the research purpose or analyzing of special structural details.

In conventional analysis and design of steel frames, beam-to-column connections are often assumed to be either fully rigid or ideally pinned for simplicity. Experiments demonstrated that connections behave nonlinearly in a manner between the two extremes of perfectly rigid and frictionless pinned connection (Popov, 1983; Tsai and Popov, 1990; Nader and Astaneh, 1991; Tsai et al., 1995). This means that there is a finite degree of joint flexibility at connections. Such connections are called semi-rigid connections. Semi-rigid connections show a significantly and major source of nonlinearity in the structural behavior of steel frames under static and dynamic loadings.
Thus, the real behavior of connections needs to be simulated accurately for analyzing the global responses of a whole structural system.

A Practical Advanced Analysis Program (PAAP) based on the refined plastic hinge method for second-order inelastic static and dynamic analysis of space steel frames was successfully developed by Kim et al (Kim et al., 2006). It was proved to be accurate and time-efficient to use in daily practical design. Thai and Kim (Thai and Kim, 2009; Thai and Kim, 2011) developed a nonlinear static algorithm, a type of truss element, and a type of cable element for nonlinear inelastic static and dynamic analysis of steel structures. However, this program ignores the effects of nonlinear beam-to-column connections and spread-of-plasticity. Therefore, the nonlinear behavior of beam-to-column connections and spread of plasticity should be included in the PAAP program for analyzing accurately the strength and deformations of steel framed structures. Another disadvantage of the PAAP program is that its solution algorithm for dynamic problems is the Newmark average accelerate method without numerical dissipation. Also, to assure the numerical stability of complex nonlinear dynamic problems, the nonlinear solution algorithm should be improved. In this research, the above-mentioned limitations of the PAAP program are overcome.

2. Objective and Scope

The overall objective of this dissertation is to develop advanced analysis methods which can capture accurately and efficiently the strength and behavior of three-dimensional semi-rigid steel framed structures subjected to static and dynamic loadings. Three nonlinear elements included both geometric and material nonlinearities are implemented into the computer program (PAAP): (1) plastic-hinge beam-column
element; (2) plastic-fiber beam-column element; and (3) multi-spring connection element. Three types of analysis provided in the proposed programs are: (1) nonlinear elastic and inelastic static analysis including rigid, linear semi-rigid, or nonlinear semi-rigid connections; (2) nonlinear elastic and inelastic time-history analysis including rigid, linear semi-rigid, or nonlinear semi-rigid connections; and (3) free vibration analysis. The proposed programs can be used to assess realistically both strength and behavior of steel framed structures and their component members in a direct manner.

Specific objectives of the research are as follows:

1. Develop a plastic-zone method using Hermite interpolation functions for second-order inelastic analysis of plane semi-rigid steel frames subjected to dynamic loadings. A computer program named NSAP (Nonlinear Structural Analysis Program) is developed.

2. Develop a plastic-fiber beam-column element using stability functions. This beam-column element is capable of capturing accurately the second-order spread-of-plasticity behavior of space steel frames.

3. Develop a general multi-spring element aiming to simulate nonlinear steel beam-to-column connections. This element can exhibit the behavior of rigid, pinned, linear semi-rigid, or nonlinear semi-rigid connections.

4. Improve the nonlinear dynamic algorithm of the PAAP program becoming the Hilber-Hughes-Taylor method (Hilber et al., 1977) because it possesses unconditional numerical stability and second-order accuracy. In addition, it can induce numerical damping in the nonlinear solution that is impossible with the Newmark average acceleration method.
The present studies are limited to steel framed structures including semi-rigid connections subjected to static and dynamic loadings. The element library of the proposed program (PAAP) is limited to “line” element including three nonlinear elements: plastic-hinge beam-column element, plastic-fiber beam-column element, and multi-spring connection element.

3. Organization of Dissertation

This dissertation includes five chapters. Contents of these chapters are as follows:

Chapter 1 introduces the motivation, objective, scope of this research and journal articles.

Chapter 2 presents a second-order distributed plasticity approach for nonlinear time-history analysis of planar semi-rigid steel frames using a developed program based on the C++ programming language (Nonlinear Structural Analysis Program - NSAP).

Chapter 3 presents a second-order plastic-hinge approach for nonlinear static and dynamic analysis of space semi-rigid steel frames using the PAAP program based on the FORTRAN 77 programming language (Practical Advanced Analysis Program – PAAP), this program is being developed by Bridge and Steel Structures Laboratory.

Chapter 4 presents a second-order spread-of-plasticity approach for nonlinear static and dynamic analysis of space semi-rigid steel frames using the PAAP program.

In Chapter 5, summary and conclusions of the present work are made and directions for future work are recommended.
THIS DISSERTATION IS SUMMARIZED FROM
JOURNAL ARTICLES


Chapter 2. SECOND-ORDER SPREAD-OF-PLASTICITY APPROACH FOR NONLINEAR DYNAMIC ANALYSIS OF TWO-DIMENSIONAL SEMI-RIGID STEEL FRAMES

1. Introduction

Conventional designs usually assume that beam-to-column connections are fully rigid or ideally pinned. This assumption causes an inaccurate prediction of the seismic response of moment-resisting steel frames because the real moment-rotation relationship of connections is a nonlinear curve, and such connections are called semi-rigid connections. Several dynamic tests were carried out to investigate the ductile and stable hysteretic behavior of steel frames, which is one of the important features of semi-rigid connections under cyclic and seismic loadings (Azizinamini and Radziminski, 1989; Nader and Astaneh, 1991; Nader and Astaneh-Asl, 1992; Elnashai and Elghazouli, 1994; Nader and Astaneh-Asl, 1996; Elnashai et al., 1998).

In order to predict actual behavior of steel frames, especially in severe loading conditions, advanced analysis methods are employed. An advanced analysis must include key factors of steel frames such as geometric nonlinearities (P – large delta and P – small delta effects), plasticity of material, nonlinear connections, geometric imperfections (out-of-straightness and out-of-plumbness), residual stress, etc., simultaneously. There are two beam-column approaches for advanced analysis of steel frame structures: (i) the plastic hinge approach (concentrated plasticity) and (ii) the distributed plasticity approach (spread-of-plasticity). In the former approach, once yielding criteria is obtained, a plastic hinge will form at one of monitored points on the
member (usually at the two ends). This method is a computationally efficient and simple way to consider the effect of inelastic material. However, the hinge methods overpredict the limit strength of structures (King et al., 1992; White, 1993; White and Chen, 1993), which can also lead to unsafe designs. What's more, it may inadequately give information as to what is happening inside the member because the member is assumed to remain fully elastic between plastic hinges. On the other hand, by the distributed plasticity approach, yielding spreads throughout the whole length and depth of members. Therefore, the distributed plasticity method is more accurate than plastic hinge methods in capturing the inelastic behavior of frame structures under severe loadings.

plasticity and nonlinear connections; both plastic hinge and refined plastic-hinge methods are presented in detail. Recently, Sekulovic and Nefovska-Danilovic (2008) (Sekulovic and Nefovska-Danilovic, 2008) applied the refined plastic hinge method and the spring-in-series concept proposed by Chan and Chui (Chan and Chui, 2000) for transient analysis of inelastic steel frames with nonlinear connections; however, their study ignored the P-small delta effects. All the above mentioned studies utilized the plastic hinge methods. Thus, analytical researches about the second-order distributed plasticity analysis of semi-rigid steel frames under dynamic loadings are uncommon.

In this paper, a sophisticated second-order spread-of-plasticity method proposed by Foley and Vinnakota (Foley and Vinnakota, 1997; Foley and Vinnakota, 1999; Foley and Vinnakota, 1999) for static analysis is developed for nonlinear inelastic time-history analysis of plane semi-rigid steel frames. An elastic-perfectly plastic model with linear strain hardening is applied to establish a new nonlinear element tangent stiffness matrix based on the principle of stationary potential energy. Accurately, to capture the second-order effects and spread of plasticity, each frame member is divided into many sub-elements along the member length and the cross-section depth. The tangent stiffness matrix of the nonlinear beam-column element directly takes into account the effects of geometric nonlinearity, gradual yielding, and flexibility of nonlinear connections. Nonlinear connections are simulated by zero-length rotational springs. The moving of the strain-hardening and elastic neutral axis, which are due to gradual yielding of the cross-section, is directly included in the element tangent stiffness matrix, and this effect is updated during the analysis process. The bowing effect, geometrical imperfections, and residual stress are also considered in this study. Three major sources of damping are
integrated in the same analysis. They are structural viscous damping, hysteretic damping due to nonlinear connections, and hysteretic damping due to material plasticity. A numerical procedure using the Newmark average acceleration method (Newmark, 1959) and the well-known Newton-Raphson iterative algorithm is proposed to solve nonlinear equations of motion. Several numerical examples are performed to illustrate the accuracy, validity, and features of the proposed second-order inelastic dynamic analysis procedure for steel frames with nonlinear flexible connections.

2. Nonlinear Finite Element Formulation

2.1 Second-Order Spread-of-Plasticity Beam-Column Element

Investigation of a typical beam-column member subjected to loads is plotted in Fig. 2.1. In order to capture the distributed plasticity, the beam-column member is divided into n elements along the member length as illustrated in Fig. 2.2; each element is divided into m small fibers within its cross section as illustrated in Fig. 2.3; and, each fiber is represented by its material properties, geometric characteristic, area $A_j$, and its coordinate location $(y_j, z_j)$ corresponding to its centroid. This way, residual stress is directly considered in assigning an initial stress value for each fiber. The second-order
effects are included by the use of several sub-elements per member through updating of the element stiffness matrix and nodal coordinates at each iterative step.

Fig. 2.2 Meshing of beam-column element into n sub-elements

To reduce the computational time when assembling the structural stiffness matrix and solving the system of nonlinear equations, n sub-elements are condensed into a typical beam-column member with the six degrees of freedom at the two ends by using the static condensation algorithm derived by Wilson (Wilson, 1974). A reverse condensation algorithm is used to find the displacements along the member length for evaluating the effects of distributed plasticity and the second-order effects.

In the development of the second-order spread-of-plasticity beam-column element, the following assumptions are made: (1) the element is initially straight and prismatic; (2) plane cross-sections remain plane after deformation and normal to the deformed axis of the element; (3) out-of-plane deformations and the effect of Poisson are neglected; (4) shear strains are negligible; (5) member deformations are small, but overall structure displacements may be large; (6) residual stress is uniformly distributed along the member length; (7) yielding of the cross-section is governed by normal stress alone; (8) the material model is linearly strain-hardening elastic-perfectly plastic; and, (9) local buckling of the fiber elements does not occur. In this study, an elastic-perfectly plastic
stress-strain relationship with linearly strain hardening used by Toma and Chen (Toma and Chen, 1992) is adopted as shown in Fig. 2.4. Strain hardening starts at the strain of \( \varepsilon_{sh} = 10\varepsilon_y \), and its modulus \( E_{sh} \) is assumed to be equal to 2\% of the elastic modulus \( E \). The total internal strain energy of a beam-column element can be expressed as follows:

\[
U = \int \int_{V_e} \sigma d\varepsilon dV
\]  

(2.1)

Fig. 2.3 Illustration of meshing of element cross-section and states of fibers

The normal stresses corresponding to the strain state of fibers are calculated as follows:

\[
\sigma = E\varepsilon \quad \text{for elastic fibers}
\]

\[
\sigma = E\varepsilon_y = \sigma_y \quad \text{for yielding fibers}
\]

\[
\sigma = E\varepsilon_y + E_{sh} (\varepsilon - \varepsilon_{sh}) = \sigma_{sh} \quad \text{for hardening fibers}
\]  

(2.2)

The total internal strain energy of a partially strain-hardening elastic-plastic beam-column element can be expanded as
Fig. 2.4 Constitutive model is assumed for steel material

where $\varepsilon$ is the normal strain at any fiber within a cross section, $\sigma$ is the normal stress at any fiber within a cross section, $E$ is the elastic modulus for the material, $E_{sh}$ is the strain-hardening modulus of the material, $V$ is the volume of fibers corresponding to their states within a cross section of an element, and subscripts $e, p(y), sh$ stand for elastic, plastic, and strain-hardening states of fiber elements, respectively. Fig. 2.3 illustrates cross-section partitions with fiber states, in which $d_{CGe}$ and $d_{CGsh}$ are the shift of the center of the initial neutral axis and the distance from the initial neutral axis to the strain-hardening neutral axis created by fibers in the strain-hardening regime, respectively.

$$U = \int_{V_e} \int_{0}^{\varepsilon} E \varepsilon \varepsilon dV_e + \int_{V_p} \left( \int_{0}^{\varepsilon} \sigma_y d\varepsilon - \int_{0}^{\varepsilon} E \varepsilon \varepsilon d\varepsilon \right) dV_p$$

$$+ \int_{V_{sh}} \left( \int_{0}^{\varepsilon} \sigma_y d\varepsilon - \int_{0}^{\varepsilon} E \varepsilon \varepsilon + \int_{\varepsilon_{sh}}^{\varepsilon} E_{sh} (\varepsilon - \varepsilon_{sh}) d\varepsilon \right) dV_{sh}$$

(2.3)
Replacing the integrations over the volume of the element in Eq. (2.3) by integrating along the length and throughout the cross section of the element, Eq. (2.3) is expressed as

\[
U = \frac{E}{2} \int_{A} \varepsilon^2 dA_{x} dx + \sigma_{y} \int_{A} \varepsilon dA_{p} dx - \frac{1}{2} \sigma_{y} \varepsilon \int_{A} dA_{p} dx + \frac{E_{sh}}{2} \int_{\text{sh}} \varepsilon^2 dA_{sh} dx \\
+ \left( \sigma_{y} - E_{sh} \varepsilon_{sh} \right) \int_{A} \varepsilon dA_{sh} dx + \frac{1}{2} \left( E_{sh} \varepsilon_{sh}^2 - \sigma_{y} \varepsilon_{sh} \right) \int_{A} \varepsilon dA_{sh} dx
\]

where \( A_{x} \) is the remaining elastic area, \( A_{p} \) is the yielding area, \( A_{sh} \) is the strain-hardening area within a cross section, and \( L \) is the length of the element.

The normal strain of the assuming beam-column element can be predicted by the following strain-displacement relationship (Goto and Chen, 1987)

\[
\varepsilon = \frac{du}{dx} \quad \text{d}v \quad \frac{1}{2} \left( \frac{dv}{dx} \right)^2
\]  

(2.5)

where \( u \) is a function describing the longitudinal displacements along the element, \( v \) is a function describing the transverse displacements, and \( dx \) is an infinitesimal length of element. In this formulation, linear shape functions and cubic Hermite shape functions are employed for longitudinal displacements and transverse displacements, respectively.

\[
\{N\}_{\text{axial}} = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix}^T
\]

(2.6)

\[
\{N\}_{\text{bend}} = \begin{bmatrix} 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} & x - \frac{2x^2}{L^2} + \frac{x^3}{L^3} & \frac{3x^2}{L^2} - \frac{2x^3}{L^3} & \frac{x^3 - x^2}{L} \end{bmatrix}^T
\]

(2.7)

Substituting Eq. (2.5) into Eq. (2.4), the total internal strain energy of the partially strain-hardening elastic-plastic beam-column element is written as
\[
U = \frac{E}{2} \int_0^L \left[ A_e \left( \frac{du}{dx} \right)^2 - 2S_{ze} \left( \frac{d^2v}{dx^2} \right) \left( \frac{du}{dx} \right) + I_{ze} \left( \frac{d^2v}{dx^2} \right)^2 \right] dx
\]

\[
+ \frac{E}{2} \int_0^L \left[ A_e \left( \frac{dv}{dx} \right)^2 - S_{ze} \left( \frac{d^2v}{dx^2} \right) \left( \frac{dv}{dx} \right)^2 + A_e \left( \frac{dv}{dx} \right)^4 \right] dx
\]

\[
+ \int_0^L \left[ P_{Ay} \left( \frac{du}{dx} \right) - M_{Ay} \left( \frac{d^2v}{dx^2} \right) + \frac{P_{Ay}}{2} \left( \frac{dv}{dx} \right)^2 \right] dx - \frac{1}{2} P_{Ay} \epsilon_y \int_0^L dx
\]

\[
+ \frac{E_{sh}}{2} \int_0^L \left[ A_{sh} \left( \frac{du}{dx} \right)^2 - 2S_{zsh} \left( \frac{d^2v}{dx^2} \right) \left( \frac{du}{dx} \right) + I_{zsh} \left( \frac{d^2v}{dx^2} \right)^2 \right] dx
\]

\[
+ \frac{E_{sh}}{2} \int_0^L \left[ A_{sh} \left( \frac{dv}{dx} \right)^2 - S_{zsh} \left( \frac{d^2v}{dx^2} \right) \left( \frac{dv}{dx} \right)^2 + A_{sh} \left( \frac{dv}{dx} \right)^4 \right] dx
\]

\[
+ (\sigma_y - E_{sh} \varepsilon_{sh}) A_{sh} \int_0^L \left( \frac{du}{dx} \right) + \frac{1}{2} \left( \frac{dv}{dx} \right)^2 \right] dx
\]

\[
- (\sigma_y - E_{sh} \varepsilon_{sh}) S_{zsh} \int_0^L \left( \frac{d^2v}{dx^2} \right) dx + \frac{1}{2} (E_{sh} \varepsilon_{sh}^2 - \sigma_y \varepsilon_y) A_{sh} \int_0^L dx
\]

(2.8)

where characteristics of the cross section illustrated in Fig. 2.3 as follows:

\[
A_e = \sum_{j=1}^{o} (A_j)_e \quad S_{ze} = d_{CGe} A_e \quad I_{ze} = \sum_{j=1}^{o} (y_j^2 A_j + I_{z,j})_e \quad (2.9)
\]

\[
A_p = \sum_{j=1}^{p} (A_j)_p \quad P_{Ay} = \sum_{j=1}^{p} (\sigma_{y,j} A_j)_p \quad M_{Ay} = \sum_{j=1}^{p} (y_j \sigma_{y,j} A_j)_p \quad (2.10)
\]

\[
A_{sh} = \sum_{j=1}^{q} (A_j)_{sh} \quad S_{zsh} = d_{CGsh} A_{sh} \quad I_{zsh} = \sum_{j=1}^{q} (y_j^2 A_j + I_{z,j})_{sh} \quad (2.11)
\]

where \(o\), \(p\), and \(q\) are the number of elastic, yielding, and strain-hardening fibers, respectively. \(I_{z,j}\) is the z-axis moment of inertia of \(j\)th fiber around its centroid, and \(d_{CGe}\) and \(d_{CGsh}\) are the shift of the center of the initial neutral axis and the distance.
from the initial neutral axis to the new strain-hardening neutral axis created by fibers in the strain-hardening state, respectively.

The potential energy of the element with loads indicated in Fig. 2.1 can be expressed as

\[
V = -\int_0^L w(x)v(x)dx - P v(P) - \{r\}^T \{d\} \tag{2.12}
\]

where \(w(x)\) is a function of the distributed load, \(P\) is the magnitude of the concentrated load, \(v(P)\) is the displacement at the position put by the concentrated load, \(\{r\}\) is the nodal load vector applied at two ends of the element, and \(\{d\}\) is the nodal displacement vector at the two ends of the element.

The total potential energy of the element is written as follows:

\[
\Pi = U + V \tag{2.13}
\]

Applying the principle of stationary potential energy, the change in the total potential energy for element vanishes for small variations in the generalized coordinates at an equilibrium configuration. In mathematical terms, this can be written as:

\[
\frac{\partial \Pi}{\partial d_i} = 0 \text{ with } i = 1, 2, \ldots 6. \tag{2.14}
\]

Taking the partial derivatives of Eq. (2.13), the set of equilibrium equations of the beam-column element can be given by

\[
\{r\} = [K_{sec}]\{d\} + \{EF\} + \{r_p\} + \{r_{sh}\}
\]

where \([K_{sec}]\) is the element secant stiffness matrix, \(\{r\}\) is the vector of element nodal forces, \(\{EF\}\) is the vector of fixed end-forces due to the superposition of both distributed and concentrated loads, \(\{r_p\}\) is the nodal load vector resisted by the yielding
area of the cross section, \( \{r_{sh}\} \) is the nodal load vector resisted by the strain-hardening area of the cross section.

The element tangent stiffness matrix can be obtained by applying a truncated Taylor series expansion of the element equilibrium equations as follows:

\[
\begin{align*}
(r_i)_{new} - (r_i)_{old} &= \frac{\partial r_i}{\partial d_j} \left[ (d_j)_{new} - (d_j)_{old} \right] \\
\Delta r_i &= \frac{\partial r_i}{\partial d_j} \Delta d_j \\
K_{T(i,j)} &= \frac{\partial r_i}{\partial d_j} \frac{\partial^2 \Pi}{\partial d_j \partial d_i} \text{ with } i, j = 1 \div 6
\end{align*}
\]

where \( [K_0] \) is a linear stiffness matrix for elastic fibers, \( [K_1] \) and \( [K_2] \) are stiffness matrices considering the second-order and bowing effects for elastic fibers, respectively, \( [K_p] \) is a plastic stiffness matrix for yielding fibers, \( [K_{sh0}] \) is a linear stiffness matrix for strain-hardening fibers, \( [K_{sh1}] \) and \( [K_{sh2}] \) are stiffness matrices considering the second-order and bowing effects for strain-hardening fibers, respectively, and \( [K_{psh}] \) is a plastic stiffness matrix for strain-hardening fibers. The component stiffness matrices of Eq. (2.17) are symmetric, and their non-zero terms can be found in Ref. (Nguyen and Kim, 2014) as follows:

Matrix \( [K_0] \)

\[
K_{0(1,1)} = \frac{EA}{L} \quad K_{0(1,3)} = -\frac{ES_{sc}}{L} \quad K_{0(1,4)} = -\frac{EA_s}{L} \quad K_{0(1,6)} = \frac{ES_{sc}}{L}
\]
\[
K_{0(2,2)} = \frac{12EI_z}{L^3} \quad K_{0(2,3)} = \frac{6EI_z}{L^3} \quad K_{0(2,5)} = -\frac{12EI_z}{L^3} \quad K_{0(2,6)} = \frac{6EI_z}{L^3} \\
K_{0(3,3)} = \frac{4EI_z}{L} \quad K_{0(3,4)} = \frac{ES_z}{L} \quad K_{0(3,5)} = -\frac{6EI_z}{L^3} \quad K_{0(3,6)} = \frac{2EI_z}{L} \\
K_{0(4,4)} = \frac{EA_x}{L} \quad K_{0(4,6)} = -\frac{ES_z}{L} \quad K_{0(5,5)} = \frac{12EI_z}{L^3} \\
K_{0(5,6)} = -\frac{6EI_z}{L^2} \quad K_{0(6,6)} = \frac{4EI_z}{L} 
\]

Matrix \([K_1]\)

\[
K_{1(1,2)} = EA_x \left[\frac{6}{5L^2} (d_5 - d_2) - \frac{1}{10L} (d_3 + d_6)\right] \quad K_{1(2,2)} = EA_x \left[\frac{6}{5L^2} (d_4 - d_1)\right] \\
K_{1(1,3)} = EA_x \left[\frac{1}{10L} (d_5 - d_2) + \frac{1}{30} (d_6 - 4d_3)\right] \quad K_{1(2,3)} = EA_x \left[\frac{1}{10L} (d_4 - d_1)\right] \\
K_{1(1,5)} = EA_x \left[-\frac{6}{5L^2} (d_5 - d_2) + \frac{1}{10L} (d_3 + d_6)\right] \quad K_{1(2,5)} = EA_x \left[-\frac{6}{5L^2} (d_4 - d_1)\right] \\
K_{1(1,6)} = EA_x \left[\frac{6}{10L} (d_5 - d_2) + \frac{1}{30} (d_3 - 4d_6)\right] \quad K_{1(2,6)} = EA_x \left[\frac{1}{10L} (d_4 - d_1)\right] \\
K_{1(2,4)} = EA_x \left[-\frac{6}{5L^2} (d_5 - d_2) + \frac{1}{10L} (d_3 + d_6)\right] \quad K_{1(3,5)} = EA_x \left[-\frac{1}{10L} (d_4 - d_1)\right] \\
K_{1(3,3)} = EA_x \left[\frac{2}{15} (d_4 - d_1)\right] + ES_z d_3 \quad K_{1(3,6)} = EA_x \left[-\frac{1}{30} (d_4 - d_1)\right] \\
K_{1(3,4)} = EA_x \left[-\frac{1}{10L} (d_5 - d_2) + \frac{1}{30} (4d_3 - d_6)\right] \quad K_{1(5,5)} = EA_x \left[\frac{6}{5L^2} (d_4 - d_1)\right] 
\]
\[ K_{i,4} = EA_e \left[ \frac{6}{5L} (d_3 - d_2) - \frac{1}{10L} (d_3 + d_6) \right] \quad K_{i,5} = EA_e \left[ -\frac{1}{10L} (d_4 - d_1) \right] \]

\[ K_{i,4} = EA_e \left[ -\frac{1}{10L} (d_3 - d_2) + \frac{1}{30} (4d_6 - d_3) \right] \]

\[ K_{i,6} = EA_e \left[ \frac{2}{15} (d_4 - d_1) \right] - ES_\sigma d_6 \]

Matrix \( [K_2] \)

\[ K_{2,2} = \frac{E_\sigma}{140} \left[ \frac{18}{L} (d_5^2 + d_6^2) + \frac{432}{L^2} (d_5 - d_2)^2 - \frac{108}{L^2} (d_5 - d_2)(d_5 + d_6) \right] \]

\[ K_{2,3} = \frac{E_\sigma}{280} \left[ 3(d_6^2 - d_5^2) + 6d_5d_6 - \frac{72}{L} d_5(d_5 - d_2) + \frac{108}{L^2} (d_5 - d_2)^2 \right] \]

\[ K_{2,5} = \frac{E_\sigma}{140} \left[ -\frac{18}{L} (d_5^2 + d_6^2) - \frac{432}{L^2} (d_5 - d_2)^2 + \frac{108}{L^2} (d_5 - d_2)(d_5 + d_6) \right] \]

\[ K_{2,6} = \frac{E_\sigma}{280} \left[ 3(d_6^2 - d_5^2) + 6d_5d_6 - \frac{72}{L} d_5(d_5 - d_2) + \frac{108}{L^2} (d_5 - d_2)^2 \right] \]

\[ K_{2,3} = \frac{E_\sigma}{140} \left[ 12Ld_5^2 + Ld_6^2 - 3Ld_5d_6 + \frac{18}{L} (d_5 - d_2)^2 - 3(d_5 - d_2)(d_6 - d_3) \right] \]

\[ K_{2,5} = \frac{E_\sigma}{280} \left[ -3(d_6^2 - d_5^2) - 6d_5d_6 + \frac{72}{L} d_5(d_5 - d_2) - \frac{108}{L^2} (d_5 - d_2)^2 \right] \]

\[ K_{2,6} = \frac{E_\sigma}{280} \left[ 4Ld_5d_6 - 3L(d_5^2 + d_6^2) - 6(d_5 - d_2)(d_3 + d_6) \right] \]

\[ K_{2,5} = \frac{E_\sigma}{280} \left[ \frac{18}{L} (d_5^2 + d_6^2) + \frac{432}{L^2} (d_5 - d_2)^2 - \frac{108}{L^2} (d_5 - d_2)(d_5 + d_6) \right] \]
\[ K_{2(5,6)} = \frac{EA_x}{280} \left[ 3(d_6^2 - d_5^2) - 6d_3d_6 + \frac{72}{L} d_6(d_5 - d_2) - \frac{108}{L^2} (d_5 - d_2)^2 \right] \]

\[ K_{2(6,6)} = \frac{EA_x}{140} \left[ \frac{18}{L} (d_5 - d_2)^2 + Ld_3^2 + 12Ld_6^2 - 3Ld_3d_6 + 3(d_5 - d_2)(d_6 - d_3) \right] \]

Matrix \[ \left[ K_p \right] \]

\[ K_{3(2,2)} = \frac{6}{5L} P_{A_p} \quad K_{3(2,3)} = \frac{1}{10} P_{A_p} \quad K_{3(2,5)} = -\frac{6}{5L} P_{A_p} \quad K_{3(2,6)} = \frac{1}{10} P_{A_p} \]

\[ K_{3(3,3)} = \frac{2L}{15} P_{A_p} \quad K_{3(3,5)} = -\frac{1}{10} P_{A_p} \quad K_{3(3,6)} = -\frac{L}{30} P_{A_p} \quad K_{3(5,5)} = \frac{6}{5L} P_{A_p} \]

\[ K_{3(5,6)} = -\frac{1}{10} P_{A_p} \quad K_{3(6,6)} = \frac{2L}{15} P_{A_p} \]

Matrix \[ \left[ K_{sh0} \right] \]

\[ K_{sh0(1,1)} = \frac{E_{sh}A_{sh}}{L} \quad K_{sh0(1,3)} = -\frac{E_{sh}S_{zsh}}{L} \quad K_{sh0(1,4)} = -\frac{E_{sh}A_{sh}}{L} \]

\[ K_{sh0(1,6)} = \frac{E_{sh}S_{zsh}}{L} \quad K_{sh0(2,2)} = \frac{12E_{sh}I_{zsh}}{L^2} \quad K_{sh0(2,3)} = \frac{6E_{sh}I_{zsh}}{L^2} \]

\[ K_{sh0(2,5)} = -\frac{12E_{sh}I_{zsh}}{L^3} \quad K_{sh0(2,6)} = \frac{6E_{sh}I_{zsh}}{L^2} \quad K_{sh0(3,3)} = \frac{4E_{sh}I_{zsh}}{L} \]

\[ K_{sh0(3,4)} = \frac{E_{sh}S_{zsh}}{L} \quad K_{sh0(3,5)} = -\frac{6E_{sh}I_{zsh}}{L^2} \quad K_{sh0(3,6)} = \frac{2E_{sh}I_{zsh}}{L} \]

\[ K_{sh0(4,1)} = \frac{E_{sh}A_{sh}}{L} \quad K_{sh0(4,3)} = -\frac{E_{sh}S_{zsh}}{L} \quad K_{sh0(4,5)} = \frac{12E_{sh}I_{zsh}}{L^3} \]
\[
K_{sh0(5,6)} = -\frac{6E_{sh} I_{zh}}{L^2} \quad K_{sh0(6,6)} = \frac{4E_{sh} I_{zh}}{L}
\]

Matrix \([K_{sh1}]\)

\[
K_{sh1(1,2)} = E_{sh} A_{sh} \left[ \frac{6}{5L^2} (d_5 - d_2) - \frac{1}{10L} (d_3 + d_6) \right]
\]

\[
K_{sh1(2,2)} = E_{sh} A_{sh} \left[ \frac{6}{5L^2} (d_4 - d_1) \right]
\]

\[
K_{sh1(1,3)} = E_{sh} A_{sh} \left[ \frac{1}{10L} (d_5 - d_2) + \frac{1}{30} (d_6 - 4d_3) \right]
\]

\[
K_{sh1(2,3)} = E_{sh} A_{sh} \left[ \frac{1}{10L} (d_4 - d_1) \right]
\]

\[
K_{sh1(1,5)} = E_{sh} A_{sh} \left[ -\frac{6}{5L^2} (d_5 - d_2) + \frac{1}{10L} (d_3 + d_6) \right]
\]

\[
K_{sh1(2,5)} = E_{sh} A_{sh} \left[ -\frac{6}{5L^2} (d_4 - d_1) \right]
\]

\[
K_{sh1(1,6)} = E_{sh} A_{sh} \left[ \frac{6}{10L} (d_5 - d_2) + \frac{1}{30} (d_6 - 4d_3) \right]
\]

\[
K_{sh1(2,6)} = E_{sh} A_{sh} \left[ \frac{1}{10L} (d_4 - d_1) \right]
\]

\[
K_{sh1(2,4)} = E_{sh} A_{sh} \left[ -\frac{6}{5L^2} (d_5 - d_2) + \frac{1}{10L} (d_3 + d_6) \right]
\]

\[
K_{sh1(3,5)} = E_{sh} A_{sh} \left[ -\frac{1}{10L} (d_4 - d_1) \right]
\]

\[
K_{sh1(3,3)} = E_{sh} A_{sh} \left[ \frac{2}{15} (d_4 - d_1) \right] + E_{sh} S_{zh} d_3
\]

\[
K_{sh1(3,4)} = E_{sh} A_{sh} \left[ -\frac{1}{10L} (d_5 - d_2) - \frac{1}{30} (d_6 - 4d_3) \right]
\]

\[
K_{sh1(3,6)} = E_{sh} A_{sh} \left[ -\frac{1}{30} (d_4 - d_1) \right]
\]

\[
K_{sh1(4,5)} = E_{sh} A_{sh} \left[ \frac{6}{5L^2} (d_5 - d_2) - \frac{1}{10L} (d_3 + d_6) \right]
\]
\[ K_{sh1(5,5)} = E_{sh} A_{sh} \left[ \frac{6}{5L} (d_4 - d_1) \right] \]
\[ K_{sh1(4,6)} = E_{sh} A_{sh} \left[ -\frac{1}{10L} (d_5 - d_2) - \frac{1}{30} (d_3 - 4d_6) \right] \]
\[ K_{sh1(5,6)} = E_{sh} A_{sh} \left[ -\frac{1}{10L} (d_4 - d_1) \right] \]
\[ K_{sh1(6,6)} = E_{sh} A_{sh} \left[ \frac{2}{15} (d_4 - d_1) \right] - E_{sh} S_{sh} d_6 \]

Matrix \[ K_{sh2} \]
\[ K_{sh2(2,2)} = \frac{E_{sh} A_{sh}}{140} \left[ \frac{18}{L} (d_3^2 + d_6^2) + \frac{432}{L^3} (d_5 - d_2)^2 - \frac{108}{L^2} (d_5 - d_2)(d_3 + d_6) \right] \]
\[ K_{sh2(2,3)} = \frac{E_{sh} A_{sh}}{280} \left[ 3(d_6^2 - d_3^2) + 6d_5 d_6 - \frac{72}{L} d_6 (d_5 - d_2) + \frac{108}{L^2} (d_5 - d_2)^2 \right] \]
\[ K_{sh2(2,5)} = \frac{E_{sh} A_{sh}}{140} \left[ -\frac{18}{L} (d_3^2 + d_6^2) - \frac{432}{L^3} (d_5 - d_2)^2 + \frac{108}{L^2} (d_5 - d_2)(d_3 + d_6) \right] \]
\[ K_{sh2(2,6)} = \frac{E_{sh} A_{sh}}{280} \left[ 3(d_3^2 - d_6^2) + 6d_5 d_6 - \frac{72}{L} d_6 (d_5 - d_2) + \frac{108}{L^2} (d_5 - d_2)^2 \right] \]
\[ K_{sh2(3,3)} = \frac{E_{sh} A_{sh}}{140} \left[ 12Ld_3^2 + Ld_6^2 - 3Ld_3 d_6 + \frac{18}{L} (d_5 - d_2)^2 - 3(d_5 - d_2)(d_6 - d_3) \right] \]
\[ K_{sh2(3,5)} = \frac{E_{sh} A_{sh}}{280} \left[ -3(d_6^2 - d_3^2) - 6d_5 d_6 + \frac{72}{L} d_6 (d_5 - d_2) - \frac{108}{L^2} (d_5 - d_2)^2 \right] \]
\[ K_{sh2(3,6)} = \frac{E_{sh} A_{sh}}{280} \left[ 4Ld_3 d_6 - 3L(d_3^2 + d_6^2) - 6(d_5 - d_2)(d_3 + d_6) \right] \]
\[ K_{sh2(5,5)} = \frac{E_{sh} A_{sh}}{140} \left[ \frac{18}{L} (d_3^2 + d_6^2) + \frac{432}{L^3} (d_5 - d_2)^2 - \frac{108}{L^2} (d_5 - d_2)(d_3 + d_6) \right] \]
\[ K_{sh2(5,6)} = \frac{E_{sh}A_{sh}}{280} \left[ 3(d_5^2 - d_3^2) - 6d_3d_6 + \frac{72}{L}d_6(d_3 - d_2) - \frac{108}{L^2}(d_5 - d_2)^2 \right] \]

\[ K_{sh2(6,6)} = \frac{E_{sh}A_{sh}}{140} \left[ \frac{18}{L}(d_5 - d_2)^2 + Ld_3^2 + 12Ld_6^2 - 3Ld_3d_6 + 3(d_5 - d_2)(d_6 - d_3) \right] \]

Matrix \[ [K_{push}] \]

\[ K_{sh3(2,2)} = \frac{6}{5L}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(2,5)} = -\frac{6}{5L}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(3,3)} = \frac{2L}{15}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(3,5)} = -\frac{L}{30}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(3,6)} = -\frac{1}{10}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(5,5)} = \frac{6}{5L}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

\[ K_{sh3(5,6)} = \frac{2L}{15}A_{sh}(\sigma_y - E_{sh}\varepsilon_{sh}) \]

2.2 Nonlinear Beam-to-Column Connections

2.2.1 Modified Tangent Stiffness Matrix including Nonlinear Connections

Fig. 2.5 Beam-column member including nonlinear connections with eight degrees of freedom
Neglecting axial and shear deformations in connections, semi-rigid connections are simulated by zero-length rotational springs attached at the two ends of the beam-column member developed above as illustrated in Fig. 2.5. The static condensation algorithm is again used to modify the beam-column member with eight degrees of freedom into the member with the conventional six degrees of freedom considering semi-rigid connections as shown in Fig. 2.6, and this also takes advantage of assembling the structural stiffness matrix. A similar procedure can be found in Ref. (Chen and Lui, 1987). A modified process is presented as follows:

Equilibrium equations of rotational spring elements are given by

\[
\begin{bmatrix}
    R_{k1} & -R_{k1} \\
    -R_{k1} & R_{k1}
\end{bmatrix}
\begin{bmatrix}
    d_3 \\
    d_7
\end{bmatrix} =
\begin{bmatrix}
    r_3 \\
    r_7
\end{bmatrix}
\]  
\tag{2.18}

\[
\begin{bmatrix}
    R_{k2} & -R_{k2} \\
    -R_{k2} & R_{k2}
\end{bmatrix}
\begin{bmatrix}
    d_6 \\
    d_8
\end{bmatrix} =
\begin{bmatrix}
    r_6 \\
    r_8
\end{bmatrix}
\]  
\tag{2.19}

where \(R_{k1}\) and \(R_{k2}\) are stiffness of rotational springs, and they are defined by the moment-rotation relationship of connections.

Equilibrium equations for the beam-column member, including nonlinear connections, with eight degrees of freedom are written as:
where $d_a$ is the condensed displacement vector including two degrees of freedom, $d_b$ is the displacement vector of the modified beam-column member with the conventional six degrees of freedom, and $P_1$, $V_1$, $M_1$ and $P_2$, $V_2$, $M_2$ are the equivalent nodal forces of a beam-column member produced by distributed and concentrated forces applied between the member ends.

Rewriting Eq. (2.21) as algebraic equations

$$[K_{aa}]\{d_a\} + [K_{ab}]\{d_b\} = \{r_a\}$$

$$[K_{ba}]\{d_a\} + [K_{bb}]\{d_b\} = \{r_b\}$$

From Eq. (2.22), we have

$$\{d_a\} = [K_{aa}]^{-1}\{r_a\} - [K_{aa}]^{-1}[K_{ab}]\{d_b\}$$

Eq. (2.24) is used to solve the condensed displacements. Substituting Eq. (2.24) into Eq. (2.23), equilibrium equations including the essential six degrees of freedom are written as
\[
\left( [K_{bb}] - [K_{ab}]^T [K_{aa}]^{-1} [K_{ab}] \right) \{d_b\} = \{r_b\} - [K_{ab}]^T [K_{aa}]^{-1} \{r_a\} \tag{2.25}
\]

\[
[K'] \{d_b\} = \{r'\} \tag{2.26}
\]

Substituting \([K_{aa}]^{-1}\) into Eq. (2.25), we obtain the modified tangent stiffness matrix \([K']\) including nonlinear connections and the modified load vector \([r']\) which are shown in Ref. (Nguyen and Kim, 2014) as follows:

Let \(\alpha_1 = (k_{33} + R_{11})\); \(\alpha_2 = (k_{66} + R_{44})\); \(\beta = \alpha_1 \alpha_2 - k_{36}^2\)

Matrix \([K_{aa}]^{-1}\) =

\[
\begin{bmatrix}
\alpha_2 & -k_{36} \\
\beta & \beta \\
-k_{36} & \alpha_1 \\
\beta & \beta
\end{bmatrix}
\]

Matrix \([K']\)

\[
K'_{(1,1)} = k_{11} - \frac{k_{13}(\alpha_2 k_{13} - k_{16} k_{36})}{\beta} - \frac{k_{16}(\alpha_4 k_{16} - k_{13} k_{36})}{\beta}
\]

\[
K'_{(1,3)} = \frac{R_{11}(\alpha_2 k_{13} - k_{16} k_{36})}{\beta}
\]

\[
K'_{(1,2)} = k_{12} - \frac{k_{23}(\alpha_2 k_{13} - k_{16} k_{36})}{\beta} - \frac{k_{26}(\alpha_4 k_{16} - k_{13} k_{36})}{\beta}
\]

\[
K'_{(1,6)} = \frac{R_{44}(\alpha_4 k_{16} - k_{13} k_{36})}{\beta}
\]

\[
K'_{(1,4)} = k_{14} - \frac{k_{34}(\alpha_2 k_{13} - k_{16} k_{36})}{\beta} - \frac{k_{46}(\alpha_4 k_{16} - k_{13} k_{36})}{\beta}
\]

\[
K'_{(2,3)} = \frac{R_{44}(\alpha_2 k_{33} - k_{26} k_{36})}{\beta}
\]

26
\[ K'_{(1,5)} = k_{15} \frac{k_{35}(\alpha_{21}k_{35} - k_{16}k_{36})}{\beta} - \frac{k_{56}(\alpha_{16}k_{16} - k_{13}k_{36})}{\beta} \]
\[ K'_{(2,6)} = \frac{R_{k2}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(2,2)} = k_{22} \frac{k_{23}(\alpha_{23}k_{23} - k_{26}k_{36})}{\beta} - \frac{k_{26}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(2,4)} = k_{24} \frac{k_{34}(\alpha_{24}k_{24} - k_{26}k_{36})}{\beta} - \frac{k_{46}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(3,4)} = \frac{R_{k4}1(\alpha_{34}k_{34} - k_{46}k_{36})}{\beta} \]
\[ K'_{(2,5)} = k_{25} \frac{k_{35}(\alpha_{25}k_{35} - k_{26}k_{36})}{\beta} - \frac{k_{56}(\alpha_{16}k_{16} - k_{13}k_{36})}{\beta} \]
\[ K'_{(3,5)} = \frac{R_{k2}(\alpha_{35}k_{35} - k_{56}k_{36})}{\beta} \]
\[ K'_{(4,4)} = k_{44} \frac{k_{34}(\alpha_{24}k_{34} - k_{46}k_{36})}{\beta} - \frac{k_{46}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(4,5)} = k_{45} \frac{k_{35}(\alpha_{45}k_{35} - k_{46}k_{36})}{\beta} - \frac{k_{56}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(4,6)} = \frac{R_{k2}(\alpha_{26}k_{26} - k_{34}k_{36})}{\beta} \]
\[ K'_{(5,5)} = k_{55} \frac{k_{35}(\alpha_{25}k_{35} - k_{56}k_{36})}{\beta} - \frac{k_{56}(\alpha_{26}k_{26} - k_{23}k_{36})}{\beta} \]
\[ K'_{(5,6)} = \frac{R_{k2}(\alpha_{26}k_{26} - k_{35}k_{36})}{\beta} \]
\[ K'_{(3,3)} = \frac{R_{k2}(\beta - R_{k2}1\alpha_{1})}{\beta} \]
\[ K'_{(3,6)} = k_{36} \frac{R_{k1}R_{k2}}{\beta} \]
\[ K'_{(6,6)} = \frac{R_{k2}(\beta - R_{k2}1\alpha_{1})}{\beta} \]
2.2.2 Moment-Rotation Relationship of Nonlinear Connections

In this study, nonlinear behavior of semi-rigid connections is represented by a nonlinear moment-rotation curve. It is expressed by a mathematical function in which the parameters are determined by the curve fitting of test results. The Richard-Abbott four-parameter model (Richard and Abbott, 1975) and the Chen-Lui exponential model (Lui and Chen, 1986) are employed for tracing nonlinear moment-rotation behavior of semi-rigid connections.

In 1975, Richard and Abbott proposed a four-parameter model (Richard and Abbott, 1975). The moment-rotation relationship of the connection is defined by

\[
M = \frac{(R_{k_2} - R_{k_p})}{\theta_0} \left| \theta_0 \right|^n + R_{k_p} \left| \theta_0 \right|^n \left( 1 + \left| \frac{(R_{k_2} - R_{k_p})}{M_0} \right|^n \right)^{-\frac{1}{n}}
\]  

(2.27)
where $M$ and $\theta_r$ are the moment and the rotation of the connection, respectively, $n$ is the shape parameter, $R_{ki}$ is the initial connection stiffness, and $R_{kp}$ is the strain-hardening stiffness and $M_0$ is the reference moment.

In 1986, Lui and Chen proposed the following exponential model (Lui and Chen, 1986):

$$M = M_0 + \sum_{j=1}^{n} C_j \left( 1 - \exp \left( \frac{w}{2\alpha^j} \right) \right) + R_{kf} \theta_r$$

(2.28)

where $M$ and $|\theta_r|$ are the moment and the absolute value of the rotational deformation of the connection, respectively, $\alpha$ is the scaling factor, $R_{kf}$ is the strain-hardening stiffness of the connection, $M_0$ is the initial moment, $C_j$ is the curve-fitting coefficient, and $n$ is the number of terms considered.

2.2.3 Cyclic Behavior of Nonlinear Connections

The independent hardening model shown in Fig. 2.7 is used to trace the cyclic behavior of nonlinear connections because of its simple application (Chen and Saleeb, 1982). The virgin $M-\theta_r$ relationship is defined by the models shown in Eqs. (2.27) or (2.28). The instantaneous tangent stiffness of connections is determined by taking the derivative $M$ on the $\theta_r$ of Eqs. (2.27) or (2.28). The hysteretic behavior of semi-rigid connections is described as follows:
1) If a connection is initially loaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows line OA with the initial stiffness $R_{ki}$ shown in Fig. 2.7, the instantaneous tangent stiffness will be $R_{si} = \frac{dM}{d[\theta_r]}$.

2) At point A, if the connection is unloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve goes back along line ABC with the initial stiffness $R_{ki}$.

3) At point C, if the connection is continuously unloaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows line CD with the initial stiffness $R_{ki}$ followed by the tangent stiffness $R_{ki}$.

4) At point D, if the connection is reloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve follows the straight line DE with the initial stiffness $R_{ki}$.

5) At point E, if the connection is continuously reloaded, the $M - \theta_r$ curve follows the line EF which is similar to line OA.

6) At point F, the connection shows a similar curve to steps 1) - 5).
3. Nonlinear Solution Procedures

A nonlinear algorithm based on the Newmark average acceleration method (Newmark, 1959) is developed for solving governing differential equations of motion because it possesses unconditional numerical stability and second-order accuracy. The incremental equation of motion of a structure can be written as

\[
[M] \{\Delta \ddot{\theta}_{t+\Delta t} \} + [C_T] \{\Delta \dot{\theta}_{t+\Delta t} \} + [K_T] \{\Delta \theta_{t+\Delta t} \} = \{\Delta F_{ext}^{t+\Delta t} \}
\]

(2.29)

where \( \{\Delta \ddot{\theta} \} \), \( \{\Delta \dot{\theta} \} \), and \( \{\Delta \theta \} \) are the vectors of incremental acceleration, velocity, and displacement, respectively; \( [M] \), \( [C_T] \), and \( [K_T] \) are mass, damping, and tangent stiffness matrices, respectively; \( \{\Delta F_{ext} \} \) is the external incremental load vector; and, superscripts \( t \) and \( t+\Delta t \) are used to distinguish the values at time \( t \) and \( t+\Delta t \). The structural viscous damping matrix \( [C_T] \) can be defined as Rayleigh damping (Chopra, 2007):
\[ [C_T] = \alpha_M [M] + \beta_K [K_T] \]  

(2.30)

where \( \alpha_M \) and \( \beta_K \) are the coefficients of mass- and stiffness-proportional damping, respectively. If both modes are assumed to have the same damping ratio \( \xi \), then

\[
\alpha_M = \xi \frac{2\omega_1\omega_2}{\omega_1 + \omega_2} ; \quad \beta_K = \xi \frac{2}{\omega_1 + \omega_2}
\]  

(2.31)

where \( \omega_1 \) and \( \omega_2 \) are the natural frequencies of the first and second modes of the frame, respectively.

Using Newmark’s approximate equations in standard form as shown in (Newmark, 1959) and using coefficients \( \gamma = \frac{1}{2} \) and \( \beta = \frac{1}{4} \), we have:

\[
\{D^{i+\Delta}\} = \{D'\} + \Delta t \{\dot{D}'\} + \left(\frac{1}{2} - \beta\right)\Delta t^2 \{\ddot{D}'\} + \beta \Delta t^2 \{\dddot{D}^{i+\Delta}\}
\]  

(2.32)

\[
\{\dot{D}^{i+\Delta}\} = \{\dot{D}'\} + (1 - \gamma) \Delta t \{\dot{D}'\} + \gamma \Delta t \{\dddot{D}^{i+\Delta}\}
\]  

(2.33)

Transforming Eqs. (2.32) and (2.33), the incremental velocity and acceleration vectors at the first iteration of each time step can be written as

\[
\{\Delta\dot{D}^{i+\Delta}\} = \frac{\gamma}{\beta \Delta t} \{\Delta D^{i+\Delta}\} - \frac{\gamma}{\beta} \{\dot{D}'\} + \left(1 - \frac{\gamma}{2\beta}\right)\Delta t \{\ddot{D}'\}
\]  

(2.34)

\[
\{\Delta\ddot{D}^{i+\Delta}\} = \frac{1}{\beta \Delta t^2} \{\Delta D^{i+\Delta}\} - \frac{1}{\beta \Delta t} \{\dot{D}'\} - \frac{1}{2\beta} \{\ddot{D}'\}
\]  

(2.35)

Substituting Eqs. (2.34) and (2.35) into Eq. (2.29), the incremental displacement vector can be calculated from

\[
[K]\{\Delta D^{i+\Delta}\} = \{\Delta F\}
\]  

(2.36)
where $\hat{K}$ and $\{\Delta \hat{F}\}$ are the effective stiffness matrix and incremental effective force vector, respectively, given as

$$
\hat{K} = [K_r] + \frac{\gamma}{\beta \Delta t}[C_r] + \frac{1}{\beta^2 \Delta t^2}[M]
$$

$$
\{\Delta \hat{F}\} = \{\Delta F^{t+\Delta t}\} + [M] \left\{ \frac{1}{\beta \Delta t} \{\dot{D}'\} + \frac{1}{2\beta} \{\ddot{D}'\} \right\} + [C_r] \left\{ \frac{\gamma}{\beta} \{\ddot{D}'\} - \left(1 - \frac{\gamma}{2\beta}\right) \Delta t \{\dddot{D}'\} \right\}
$$

Unbalanced forces in each time step can be eliminated by using the well-known Newton-Raphson iterative method. At the first iteration of each time step, the total displacement, velocity and acceleration at the time $t + \Delta t$ are updated based on the incremental displacement $\{\Delta D^{t+\Delta t}\}$ as follows:

$$
\{D^{t+\Delta t}\} = \{D'\} + \{\Delta D^{t+\Delta t}\}
$$

$$
\{\dot{D}^{t+\Delta t}\} = \left(1 - \frac{\gamma}{2\beta}\right) \Delta t \{\ddot{D}'\} + \left(1 - \frac{\gamma}{\beta}\right) \{\ddot{D}'\} + \frac{\gamma}{\beta \Delta t} \{\Delta D^{t+\Delta t}\}
$$

$$
\{\ddot{D}^{t+\Delta t}\} = \left(1 - \frac{1}{2\beta}\right) \{\dddot{D}'\} - \frac{1}{\beta \Delta t} \{\dddot{D}'\} + \frac{1}{\beta^2 \Delta t^2} \{\Delta D^{t+\Delta t}\}
$$

For the second and subsequent iterations of each time step, the structural system is solved under the effect of the unbalanced force vector $\{R\}$ as

$$
\hat{K}_k \{\delta \Delta D^{t+\Delta t}\}_{k+1} = \{R\}_k
$$

where the effective stiffness matrix $\hat{K}_k$ and the residual force vector $\{R\}_k$ are calculated at the unbalanced iterative step $k$, respectively, as follows:
\[
\begin{align*}
\left[ \dot{K} \right]_k &= \left[ K_r \right]_k + \frac{\gamma}{\beta \Delta t} \left[ C_r \right] + \frac{1}{\beta \Delta t^2} \left[ M \right] \\
\{ R \}_k &= \{ F_{\text{ext}}^{t+\Delta t} \} - \{ F_{\text{int}} \}_k - \{ F_{\text{dam}} \}_k - \{ F_{\text{inc}} \}_k
\end{align*}
\] (2.43)

where \( \{ F_{\text{ext}}^{t+\Delta t} \} \) is the total external force vector. The inertial force vector \( \{ F_{\text{inc}} \}_k \), the damping force vector \( \{ F_{\text{dam}} \}_k \), and the updated internal force vector \( \{ F_{\text{int}} \}_k \) at the unbalanced iterative step \( k \) are respectively defined as:

\[
\{ F_{\text{inc}} \}_k = \left[ M \right] \{ \dot{D}^{t+\Delta t} \}_k \\
\{ F_{\text{dam}} \}_k = \left[ C_r \right] \{ \dot{D}^{t+\Delta t} \}_k \\
\{ F_{\text{int}} \}_k = \{ F_{\text{int}} \} \left( \{ D^{t+\Delta t} \}_k \right)
\] (2.44)

At each iterative step, the state of each fiber, characteristic of a cross section of each beam-column element, stiffness of semi-rigid springs are updated for assembling the new structural stiffness matrix. Once the convergence criterion is satisfied, the structural response history is saved for the next time step as

\[
\{ \Delta D^{t+\Delta t} \}_k = \{ \Delta D^{t+\Delta t} \}_k + \{ \Delta \Delta D^{t+\Delta t} \}_k
\] (2.45)

\[
\{ D^{t+\Delta t} \}_k = \{ D^{t+\Delta t} \}_k = \{ \dot{D}^t \} + \{ \Delta D^{t+\Delta t} \}_k
\] (2.46)

\[
\{ \dot{D}^{t+\Delta t} \}_k = \{ \dot{D}^{t+\Delta t} \}_k = \left( 1 - \frac{\gamma}{2\beta} \right) \Delta t \{ \ddot{D}^t \} + \left( 1 - \frac{\gamma}{\beta} \right) \{ \dot{D}^t \} + \frac{\gamma}{\beta \Delta t} \{ \Delta \Delta D^{t+\Delta t} \}_k
\] (2.47)

\[
\{ \ddot{D}^{t+\Delta t} \}_k = \{ \ddot{D}^{t+\Delta t} \}_k = \left( 1 - \frac{1}{2\beta} \right) \{ \dddot{D}^t \} - \frac{1}{\beta \Delta t} \{ \ddot{D}^t \} + \frac{1}{\beta \Delta t^2} \{ \Delta \Delta D^{t+\Delta t} \}_k
\] (2.48)
4. Numerical Examples and Discussions

A computer program written in the C++ programming language named Nonlinear Structural Analysis Program (NSAP) is developed based on the above-mentioned formulations to predict second-order spread-of-plasticity time-history responses of plane steel frames with nonlinear beam-to-column connections subjected to static loads, earthquakes, and dynamic-force loadings. It is verified for accuracy and efficiency by the comparison of the predictions with those generated by ABAQUS and previous studies in the literature. The isotropic hardening model for cyclic behavior is applied for steel material. All the frame members are divided into forty discrete elements. Each cross-section of elements is divided into sixty six fibers (twenty seven at each flange, twelve at the web) as shown in Fig. 2.3. Earthquake records as shown in Fig. 2.8 are used as ground excitation in dynamic analysis. Their peak ground accelerations and time steps are listed in Table 2.1.

Table 2.1 Peak ground acceleration and its corresponding time steps of earthquake records (PEER, 2011)

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>PGA (g)</th>
<th>Time step (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro (1940)</td>
<td>0.319</td>
<td>0.020</td>
</tr>
<tr>
<td>(Array, #9, USGS Station 117)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loma Prieta (1989)</td>
<td>0.529</td>
<td>0.005</td>
</tr>
<tr>
<td>(Capitola, 000, CDMG Station 47125)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northridge (1994)</td>
<td>0.640</td>
<td>0.010</td>
</tr>
<tr>
<td>(Simi Valley-Katherine, 090, USC Station 90055)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>San Fernando (1971)</td>
<td>1.160</td>
<td>0.010</td>
</tr>
<tr>
<td>(Pacoima Dam, 254, CDMG Station 279)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(a) El Centro

(b) Loma Prieta
Fig. 2.8. Earthquake records

(c) Northridge

(d) San Fernando

Fig. 2.8. Earthquake records
4.1 Portal Steel Frame subjected to Earthquakes

Fig. 2.9. Portal frame subjected to earthquakes

Fig. 2.9 illustrates the geometry and material properties of a portal frame with masses lumped at the frame nodes. In the numerical modeling, each frame member is modeled by using forty discrete elements in both the proposed program and ABAQUS (using B22 Timoshenko beam element) since the analysis cannot accurately capture the nonlinear inelastic response of the frame if only a few elements per member are used in the modeling. All elements are divided into sixty-six fibers (twenty seven at both flanges, twelve at the web) on the cross section in the proposed program.

After performing the vibration analysis, the first two natural periods along the applied earthquake direction of the portal frame are obtained and compared in Table 2.2. It can be seen that a strong agreement of natural periods of the frame generated by ABAQUS and the proposed program is obtained. These two natural periods are used to estimate the Rayleigh damping matrix by assuming the equivalent viscous damping ratio $\xi$ of 5% in the next time-history analysis step.

\[
\begin{align*}
M &= 10 \text{ Ns}^2/\text{mm} \\
E &= 200,000 \text{ MPa} \\
\nu &= 0.3 \\
\sigma_y &= 300 \text{ MPa}
\end{align*}
\]
Table 2.2 Comparison of first two natural periods (s) along the applied earthquake direction of portal frame

<table>
<thead>
<tr>
<th>Mode</th>
<th>ABAQUS</th>
<th>Present</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8162</td>
<td>0.8181</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>0.0290</td>
<td>0.0291</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 2.3 Comparison of peak displacements (mm) of portal frame

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Max/Min</th>
<th>Analysis type</th>
<th>ABAQUS</th>
<th>Present</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loma Prieta</td>
<td>Max</td>
<td>Elastic</td>
<td>104.47</td>
<td>104.42</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>104.76</td>
<td>105.23</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-88.23</td>
<td>-89.67</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-84.46</td>
<td>-83.12</td>
<td>-1.59</td>
</tr>
<tr>
<td>San Fernando</td>
<td>Max</td>
<td>Elastic</td>
<td>119.52</td>
<td>116.07</td>
<td>-2.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>122.50</td>
<td>119.27</td>
<td>-2.64</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-93.51</td>
<td>-88.86</td>
<td>-4.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-79.07</td>
<td>-78.25</td>
<td>-1.03</td>
</tr>
</tbody>
</table>

Without considering the initial residual stress, the second-order elastic and second-order inelastic displacement history of the frame under two different earthquakes of Loma Prieta and San Fernando are compared in Fig. 2.10 and Fig. 2.11, respectively. A comparison of the peak displacements is given in Table 2.3 with the maximum difference of 4.98%. It can be observed that the proposed program and ABAQUS generate nearly identical results in all cases, including the permanent drifts of displacement due to the gradual yielding behavior in the second-order inelastic analysis cases. The discrepancy in displacement response of the second-order elastic (SE) and second-order inelastic (SI) analyses is relatively clear.
In considering the effect of initial ECCS residual stress (Kim, Ngo-Huu and Lee, 2006), Fig. 2.12 shows comparisons of the second-order inelastic responses of the frame. It can be seen that the permanent drifts of the roof floor have not significantly differed in the two cases. However, the permanent plastic deformation of the fiber no. 1 in the cut section A-A illustrated in Fig. 2.13 has significant differences as plotted in Fig. 2.14 and Fig. 2.15.
Fig. 2.10. Displacement time-history responses of portal frame under Loma Prieta earthquake

(a) Second-order elastic

(b) Second-order inelastic
Fig. 2.11. Displacement time-history responses of portal frame under San Fernando earthquake

(b) Second-order inelastic

(a) Loma Prieta
(b) San Fernando

Fig. 2.12. Second-order inelastic responses of portal frame with and without residual stress

Fig. 2.13. Cut section A-A and fiber no. 1 is being monitored
Using the same personal computer configuration (AMD Phenom II X4 955 Processor, 3.2 GHz; 4.00 GB RAM), the analysis time of the proposed program and ABAQUS for the second-order inelastic responses of the frame subjected to San Fernando earthquake are 1min 26sec and 25min 52sec, respectively. The analysis time of ABAQUS is 18 times longer than the proposed program. This result demonstrates the higher computational efficiency of the proposed program.

(a) Loma Prieta
Fig. 2.14. Plastic deformation and stress at fiber no. 1 in cut section A-A in portal frame with and without residual stress.
Fig. 2.15. Plastic deformation and stress at fiber no. 1 in cut section A-A in portal frame with and without residual stress

4.2 Two-Story Steel Frame with Nonlinear Connections

A single-bay two-story steel frame with flexible beam-to-column connections was studied by Chan and Chui (Chan and Chui, 2000). The geometry and loading of the frame are given in Fig. 2.16. All the frame members are W8x48 with Young’s modulus $E$ of $205 \times 10^6$ kN/m$^2$, yielding stress $\sigma_y$ of 235 MPa, and initial ECCS residual stress distribution (Kim, Ngo-Huu and Lee, 2006). An initial geometric imperfection of column $\psi$ of 1/438 is considered. The vertical static loads are applied on the frame to consider the second-order effects, and then the horizontal forces are applied suddenly at each floor during 0.5 sec, as shown in Fig. 2.16. The lumped masses of 5.1 and 10.2 Ton are modeled at the top of columns and the middle of the beams, respectively. A
time step $\Delta t$ of 0.001 sec is chosen and viscous damping of structure is ignored. The four parameters of the Richard-Abbott model for beam-to-column connections are: $R_{ki} = 23,000 \text{kN} \cdot \text{m/ rad}$, $R_{kp} = 70 \text{kN} \cdot \text{m/ rad}$, $M_o = 180 \text{kN} \cdot \text{m}$, and $n = 1.6$.

![Diagram of a two-story steel frame with nonlinear connections](image)

The second-order elastic time-history responses predicted by the proposed program for the rigid, linear semi-rigid, and nonlinear semi-rigid frames match well with those of Chan and Chui (Chan and Chui, 2000) as shown in Fig. 2.17 and Fig. 2.19a. In the case of the second-order inelastic responses, as shown in Fig. 2.18 and Fig. 2.19b, it can be recognized that the differences are due to the modeling of plasticity. The proposed program captures spread of plasticity, whereas Chan and Chui’s study uses the concentrated plastic hinge method. The moment-rotation relationships at connection C are also plotted in Fig. 2.19 for both second-order elastic and inelastic analyses.
(a) Rigid connections

(b) Linear semi-rigid connections
Fig. 2.17. Second-order elastic responses of two-story frame for various connections
Fig. 2.18. Second-order inelastic responses of two-story frame for various connections.
Fig. 2.19. Hysteresis loops at connection C of two-story frame
4.3 Vogel Six-Story Steel Frame with Nonlinear Connections – A Case Study

Vogel (Vogel, 1985) presented a two-bay six-story steel frame as a calibration frame for second-order inelastic static analysis. Chui and Chan (Chui and Chan, 1996) built the semi-rigid beam-to-column joints to study the dynamic behavior involving the

---

Vogel Six-Story Steel Frame with Nonlinear Connections – A Case Study

- Lumped mass due to vertical distributed static load.
- Semi-rigid connection.
- $F_1(t) = 10.23 \sin(\omega t) \text{kN}$
- $F_2(t) = 20.44 \sin(\omega t) \text{kN}$
- $E = 205 \times 10^6 \text{kN/m}^2$
- $\rho = 7.8 \text{T/m}^3$
- $\psi = 1/450$

Fig. 2.20. Vogel six-story steel frame with semi-rigid connections

Vogel (Vogel, 1985) presented a two-bay six-story steel frame as a calibration frame for second-order inelastic static analysis. Chui and Chan (Chui and Chan, 1996) built the semi-rigid beam-to-column joints to study the dynamic behavior involving the
connection flexibility, as shown in Fig. 2.20. An initial out-of-plumbness $\psi$ of $1/450$ was assigned for all the column members. Young’s modulus was $205 \times 10^6$ kN/m$^2$, and viscous damping was ignored. The curve fitted parameters of the Chen-Lui exponential model for a flush end plate connection were as follows: $R_{\alpha} = 12340.198$ kN.m/rad, $R_{\gamma} = 108.924$ kN.m/rad, $M_o = 0.0$ kN.m, $\alpha = 0.00031783$, $C_1 = -28.286$, $C_2 = 573.189$, $C_3 = -3433.98$, $C_4 = 8511.3$, $C_5 = -9362.567$, and $C_6 = 3832.899$ (unit of $C_i$ is kN.m) (Ostrander, 1970). The static loads of 31.7 and 49.1 kN/m$^2$ uniformly distributed on beams and the self-weight density of 7.8 kN/m$^3$ were converted to lumped masses at the frame joints. The fundamental natural frequencies for the cases of fully rigid and linear semi-rigid connections were found to be 2.41 rad/sec and 1.66 rad/sec, respectively (Chui and Chan, 1996). The frame with various types of connections under different horizontal loads, $F_1(t)$ and $F_2(t)$, is investigated.

Table 2.4 Periods and Rayleigh damping coefficients of Vogel frame

<table>
<thead>
<tr>
<th>Frame types</th>
<th>1st period (s)</th>
<th>2nd period (s)</th>
<th>$\xi$</th>
<th>$\alpha_M$</th>
<th>$\beta_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>2.6108</td>
<td>1.0116</td>
<td>0.05</td>
<td>0.1735</td>
<td>0.0116</td>
</tr>
<tr>
<td>Linear semi-rigid</td>
<td>3.7662</td>
<td>1.2846</td>
<td>0.05</td>
<td>0.1244</td>
<td>0.0152</td>
</tr>
</tbody>
</table>

Ignoring the bowing matrix $[K_2]$ in Eq. (2.17), the accuracy of the proposed program in the second-order elastic response analysis of the various frames under dynamic loadings with $\omega = 1.66$ and $2.41$ rad/sec is illustrated in Fig. 2.21 by comparing with results of Chan and Chui (Chui and Chan, 1996) without bowing effects. It can be seen that the presented results are strongly identical with those of Chan and Chui. It is noted that resonances are observed for the linear semi-rigid and rigid frames shown in
Fig. 2.21 when the frequency of the forced loadings is equal to the fundamental natural frequency of the frames. Resonance phenomenon does not occur with the frame including nonlinear connections because of existence of hysteretic damping through hysteresis loops in the connections. In case of including the bowing matrix, the second-order elastic displacement responses of the linear semi-rigid frame and the rigid frame under the dynamic loadings with $\omega = 1.66$ and 2.41 rad/sec, respectively, are different with those without bowing effects as shown in Fig. 2.22. It can be concluded that the bowing effect amplifies deflection of frames when their displacements are adequately large. The nonlinear moment-rotation response at connection C generated by the proposed program also agrees with that of Chan and Chui, as plotted in Fig. 2.23.

![Graph showing lateral displacement vs. time for different connections and load frequencies.](image)

(a) $\omega = 1.66$ rad/sec
Fig. 2.21. Second-order elastic displacement responses at roof floor of Vogel frame under forced loadings – without bowing effects.

(a) $\omega = 1.66$ rad/sec

(b) $\omega = 2.41$ rad/sec
Fig. 2.22. Second-order elastic displacement responses at roof floor of Vogel frame under forced loadings – with bowing effects

(b) $\omega = 2.41$ rad/sec

Fig. 2.23. Moment-rotation responses at connection C of Vogel frame under forced loadings with $\omega = 1.66$ rad/sec in the second-order elastic analysis
(a) Rigid connections

(b) Linear connections
Fig. 2.24. Second-order inelastic displacement responses at roof floor of Vogel frame under El Centro earthquake considering geometric imperfections

(c) Nonlinear connections

Fig. 2.25. Moment-rotation responses at connection C of Vogel frame under El Centro earthquake considering geometric imperfections in the second-order inelastic analysis
In this case study, the combined effect of inelastic hysteretic damping due to spread of plasticity, hysteresis loops of semi-rigid connections, residual stress, and initial geometric imperfections acting on overall structural responses is investigated. Yielding stress of 300 Mpa and initial residual stress of ECCS (Kim, Ngo-Huu and Lee, 2006) are used. Initial member out-of-straightness is considered by employing the reduced tangent modulus method proposed by (Kim and Chen, 1996) and (Chen and Kim, 1997). In the proposed program, Young’s modulus of $0.85 \times E$ is directly assigned for all steel members to consider initial member out-of-straightness. Before performing the second-order inelastic dynamic analysis, the frames are fully loaded by the distributed loadings on the beams. Viscous damping of the Rayleigh type is utilized, and its coefficients are presented in Table 2.4. The frame with various connections subjected to the El Centro earthquake (shown in Fig. 2.8a and Table 2.1) is analyzed for four cases of geometric imperfections (case 1 – without residual stress and initial member out-of-straightness, case 2 – considering only residual stress, case 3 – considering only initial member out-of-straightness, and case 4 – considering both residual stress and initial member out-of-straightness). As shown in Fig. 2.24, the second-order inelastic dynamic responses of the frame with various connections are clearly different. No significant differences in the second-order inelastic dynamic responses are observed between the frame models that include the initial residual stress and those without this effect, whereas the initial member out-of-straightness strongly acts on final behavior of the frame from 5th to 30th sec, as shown in Fig. 2.24. It can be concluded that the initial member out-of-straightness has much greater impact on the frame behavior than the effect of residual stress. Fig. 2.25 plots nonlinear moment-rotation responses at connection C.
corresponding to the four cases. It can be seen that hysteresis loops are unstable due to member-force redistribution caused by gradual yielding of framed members.

5. Summary and Conclusions

An accurate numerical procedure is presented for the second-order spread-of-plasticity analysis of plane steel frames under dynamic and seismic loadings. By assuming that steel behaves as an elastic-perfectly plastic material with linear strain hardening, a new nonlinear beam-column stiffness matrix including flexibility of nonlinear connections is shown in this study. The effects of gradual yielding, geometric nonlinearity, connection flexibility, bowing, moving of the neutral axis, initial member out-of-straightness, and residual stress can be directly taken into account through the element tangent stiffness matrix. Three major sources of damping are considered. They are structural viscous damping, hysteretic damping due to gradual yielding of material, and hysteretic damping due to hysteresis loops of nonlinear connections. The accuracy and efficiency of the proposed procedure are proved by comparing the results with the commercial finite element package ABAQUS and previous studies. The following conclusions can be drawn from the present study:

- The flexibility of nonlinear semi-rigid connections plays a major role in the overall structural responses during dynamic and seismic loadings.
- Hysteretic damping created by energy dissipation of nonlinear semi-rigid connections helps eliminate resonance which amplifies time-history responses of framed structures.
- Residual stress might not cause significant influence on second-order inelastic
time-history behavior of steel frames, whereas initial geometric imperfections clearly change the response of steel frames.

- It is necessary to include initial geometric imperfections and connection flexibility into advanced analysis methods to increase accuracy and safety for performance-based seismic designs of steel frames.

- The accurate results obtained in a short analysis time prove that the proposed program can be effectively used in predicting second-order inelastic time-history behavior of plane steel frames instead of using the time-consuming commercial finite element analysis software.

- The presented numerical examples can be used to verify the validity and accuracy of simple practical advanced analysis methods as benchmarks.

- The proposed algorithm can be also used to develop three-dimensional advanced analysis programs for framed structures subjected to dynamic and seismic loadings.
Chapter 3. SECOND-ORDER PLASTIC-HINGE APPROACH FOR NONLINEAR STATIC AND DYNAMIC ANALYSIS OF THREE-DIMENSIONAL SEMI-RIGID STEEL FRAMES

1. Introduction

Steel moment resisting frames have been extensively used in areas of high seismic risk for low and mid-rise buildings due to their high ductility. Conventional analysis and design of steel framed structures are usually conducted under the assumption that the beam-to-column connections are either fully rigid or frictionless pinned joints. However, in the 1994 Northridge and 1995 Kobe earthquakes, these structures, especially fully welded connections, were heavily and unexpectedly damaged. Since then, over the last three decades, several experimental and analytical studies have been conducted to investigate the dynamic behavior of alternative connection types, including semi-rigid connections.

The results of experimental studies showed that semi-rigid steel frames feature the ductile and stable hysteretic behavior of the frames when the connections are designed appropriately (Azizinamini and Radziminski, 1989; Nader and Astaneh, 1991; Elnashai and Elghazouli, 1994; Elnashai, Elghazouli and Denesh-Ashtiani, 1998). The energy is dissipated through hysteresis loops of semi-rigid connections, which are one of the important damping sources of structures. Those results also showed that if the connection stiffness increases the base shear increases but the lateral drift does not proportionally decrease. Base on the impressive characteristics of semi-rigid connections, several mathematic models were proposed to represent actual behavior of
these connections. These models can be grouped into two categories: linear connection models (Yu and Shanmugam, 1988; Xu, 2002) and nonlinear connection models (Ramberg and Osgood, 1943; Richard and Abbott, 1975; Lui and Chen, 1986; Chen and Kishi, 1989). The linear connection models can not properly predict the connection behavior due to the stiffness of the connection being assumed to be constant during the analysis procedure; while the moment-rotation curve of connections is captured more exactly by the nonlinear connection models which fitted well in the experimental curve.

In recent years, numerous analytical studies have been conducted to investigate nonlinear inelastic dynamic behavior of semi-rigid frames under dynamic and seismic loadings by Lui and Lopes (Lui and Lopes, 1997), Awkar and Lui (Awkar and Lui, 1999), and Sekulovic and Nefovska (Sekulovic and Nefovska-Danilovic, 2008), among others. In the studies (Lui and Lopes, 1997) and (Awkar and Lui, 1999), the effects of geometric nonlinearity were considered by using stability functions, the material nonlinearity was considered by refined plastic hinge approach, and the connection nonlinearity was considered by modifying the stiffness matrix of beam-column element functions that account for finite rotation at the two ends of elements. Sekulovic and Nefovska (Sekulovic and Nefovska-Danilovic, 2008) presented a spring-in-series model considering both the material and connection nonlinearities, while the geometric nonlinearity was considered by using the geometric stiffness matrix obtained from approximate functions. To capture exactly the effects of geometric nonlinearity, the members need to be divided into several elements so that it consumes analysis time significantly. Though these studies considered all three nonlinear components in the same analysis, they are limited to planar semi-rigid steel frames. With the following
procedure, all limitations of the above-mentioned studies are overcome by the integration of three nonlinearities in the complex time-history analysis of three-dimensional frames.

This study aims to investigate the simultaneous effects of three nonlinear components on the static and dynamic behavior of different steel-frame types under various static and dynamic loadings. The geometric nonlinearity caused by the interaction between the axial force and bending moments is considered by the use of stability functions, which can accurately capture the $P-\delta$ effect by simulating only one element per member; the $P-\Delta$ effect is considered by the use of the geometric stiffness matrix. The material nonlinearity is accounted for by using the refined plastic hinge method (Kim et al., 2001), in which the Column Research Council (CRC) tangent modulus concept is used to account for the gradual yielding due to residual stresses, while the gradual yielding due to flexure is represented by a parabolic function combined with the yielding surface proposed by Orbison (Orbison et al., 1982) or LRFD. An independent zero-length connection element with six different translational and rotational springs connecting two identical nodes is developed to simulate the beam-to-column connections. This is an efficient way because the modification of the beam-column stiffness matrix considering the semi-rigid connections is not necessary and the connection is ready to integrate with any beam-column model. The independent hardening model is used for considering cyclic behavior of rotational springs through employing the static mathematic models (Kishi-Chen (Chen and Kishi, 1989), Richard-Abbott (Richard and Abbott, 1975), and Chen-Lui (Lui and Chen, 1986)).
This study is based on assumptions as follows: warping torsion and axial shortening due to member curvature bending (bowing effects) are ignored; lateral-torsion buckling of members is assumed to be prevented by adequate lateral braces; a compact W-section is assumed so that the section can develop full plastic moment capacity without local buckling; the connection element length is equal to zero; a possible joint degradation due to actions of cyclic loading is not considered.

In the nonlinear static analysis, the Generalized Displacement Control method (GDC) is applied to solve the nonlinear equilibrium equations in an incremental-iterative scheme. The nonlinear response results are compared with those of existing studies to verify the accuracy of the proposed numerical procedure. In the nonlinear time-history analysis, an incremental-iterative scheme based on the Hilbert-Hughes-Taylor method and the Newton-Raphson method was developed for solving the nonlinear equations of motion. Viscous damping accounts for the use of Rayleigh damping matrix. Several examples are presented to verify the accuracy and efficiency of the proposed numerical procedure in predicting the nonlinear dynamic response of three-dimensional framed structures with semi-rigid connections.

2. Nonlinear Finite Element Formulation

2.1 Second-Order Plastic-Hinge Beam-Column Element

2.1.1 Stability Functions accounting for Second-Order Effects

To capture the effect of the interaction between axial force and bending moment, the stability functions are used to minimize modeling and solution time. Generally only one element per member is needed to capture the $P-\delta$ effect accurately. From Kim et al.
(Kim, Park and Choi, 2001), the incremental form of member basic force and deformation relationship of three-dimensional beam-column element can be expressed as

\[
\begin{pmatrix}
P \\ M_{yA} \\ M_{yB} \\ M_{zA} \\ M_{zB} \\ T
\end{pmatrix} =
\begin{pmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & S_{1y} \frac{EI_{y}}{L} & S_{2y} \frac{EI_{y}}{L} & 0 & 0 & 0 \\
0 & S_{1z} \frac{EI_{z}}{L} & S_{2z} \frac{EI_{z}}{L} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{1c} \frac{EI_{c}}{L} & S_{2c} \frac{EI_{c}}{L} & 0 \\
0 & 0 & 0 & S_{2z} \frac{EI_{z}}{L} & S_{1z} \frac{EI_{z}}{L} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{GJ}{L}
\end{pmatrix}
\begin{pmatrix}
\delta \\
\theta_{yA} \\
\theta_{yB} \\
\theta_{zA} \\
\theta_{zB} \\
\phi
\end{pmatrix}
\]  

(3.1)

where \( P \), \( M_{yA} \), \( M_{yB} \), \( M_{zA} \), \( M_{zB} \), and \( T \) are incremental axial force, end moments with respect to \( y \) and \( z \) axes, and torsion, respectively; \( \delta \), \( \theta_{yA} \), \( \theta_{yB} \), \( \theta_{zA} \), \( \theta_{zB} \), and \( \phi \) are the incremental axial displacement, the joint rotations, and the angle of twist; \( A \), \( I_{y} \), \( I_{z} \), \( J \) and \( L \) are area, moment of inertia with respect to \( y \) and \( z \) axes, torsional constant, and length of beam-column element; \( E \) and \( G \) are elastic and shear modulus of material; \( S_{1n} \) and \( S_{2n} \) are the stability functions with respect to the \( n \) axis (\( n = y, z \)) given by Chen and Lui (Chen and Lui, 1987) as

\[
S_{1n} = \begin{cases} 
\frac{\pi \sqrt{\rho_n} \sin(\pi \sqrt{\rho_n}) - \pi^2 \rho_n \cos(\pi \sqrt{\rho_n})}{2 - 2 \cos(\pi \sqrt{\rho_n}) - \pi \sqrt{\rho_n} \sin(\pi \sqrt{\rho_n})} & \text{if } P < 0 \\
\frac{\pi^2 \rho_n \cosh(\pi \sqrt{\rho_n}) - \pi \sqrt{\rho_n} \sinh(\pi \sqrt{\rho_n})}{2 - 2 \cosh(\pi \sqrt{\rho_n}) + \pi \sqrt{\rho_n} \sinh(\pi \sqrt{\rho_n})} & \text{if } P > 0
\end{cases}
\]  

(3.2)
The solutions obtained from Eqs. (3.2-3.3) are indeterminate when the axial force is zero. To overcome this problem and to avoid the use of different expressions for $S_{1n}$ and $S_{2n}$ for a different sign of axial force, Lui and Chen (Lui and Chen, 1986) have proposed a set of expressions that make use of power-series expansions to approximate the stability functions. The power-series expressions have been shown to converge to a high degree of accuracy within the first ten terms of the series expansions. Alternatively, if the axial force in the member falls within the range $-2.0 < \rho_n < 2.0$ ($n = y, z$), the following simplified expressions may be used to closely approximate the stability functions

$$S_{1n} = 4 + \frac{2\pi^2 \rho_n}{15} - \frac{(0.01 \rho_n + 0.543) \rho_n^2}{4 + \rho_n} + \frac{(0.004 \rho_n + 0.285) \rho_n^2}{8.183 + \rho_n}$$

(3.4)

$$S_{2n} = 2 - \frac{\pi^2 \rho_n}{30} + \frac{(0.01 \rho_n + 0.543) \rho_n^2}{4 + \rho_n} - \frac{(0.004 \rho_n + 0.285) \rho_n^2}{8.183 + \rho_n}$$

(3.5)

For most practical applications, it gives an excellent correlation to the "exact" expressions given by Eqs. (3.2-3.3). However, for $\rho_n$ other than the range $-2.0 \leq \rho_n \leq 2.0$, the conventional stability functions in Eqs. (3.2-3.3) should be used.
2.1.2 Refined Plastic Hinge Model accounting for inelastic effects

The material nonlinearity includes gradual yielding of steel associated with residual stresses and flexure. The gradual yielding due to residual stresses is considered by utilizing the Column Research Council (CRC) tangent modulus concept $E_t$, while the gradual yielding due to flexure is represented by a parabolic function. The relationship between the basic force and deformation of 3-D beam-column is modified to account for the inelastic effects as

$$\begin{bmatrix} P \\ M_{yA} \\ M_{yB} \\ M_{zA} \\ M_{zB} \\ T \end{bmatrix} = \begin{bmatrix} E_t A/L & 0 & 0 & 0 & 0 \\ 0 & k_{ijy} & k_{ijy} & 0 & 0 \\ 0 & k_{ijy} & k_{ijy} & 0 & 0 \\ 0 & 0 & 0 & k_{ijz} & k_{ijz} \\ 0 & 0 & 0 & k_{ijz} & k_{ijz} \\ 0 & 0 & 0 & 0 & GJ/L \end{bmatrix} \begin{bmatrix} \delta \\ \theta_{yA} \\ \theta_{yB} \\ \theta_{zA} \\ \theta_{zB} \\ \phi \end{bmatrix}$$

(3.6)

where

$$k_{ijy} = \eta_A (S_1 - \frac{S_2^2}{S_1^2} (1 - \eta_B)) \frac{E_t I_y}{L}$$

(3.7)

$$k_{ijy} = \eta_A \eta_B S_2 \frac{E_t I_y}{L}$$

(3.8)

$$k_{ijy} = \eta_B (S_1 - \frac{S_2^2}{S_1^2} (1 - \eta_A)) \frac{E_t I_y}{L}$$

(3.9)

$$k_{ijz} = \eta_A (S_2 - \frac{S_2^2}{S_2^2} (1 - \eta_B)) \frac{E_t I_z}{L}$$

(3.10)

$$k_{ijz} = \eta_A \eta_B S_2 \frac{E_t I_z}{L}$$

(3.11)
From (Chen and Lui, 1987), the CRC tangent modulus $E_t$ accounting for the effects of residual stresses and gradual yielding due to axial force is written as

$$E_t = 1.0 \frac{P}{P_y} \text{ for } P \leq 0.5 P_y$$  \hspace{1cm} (3.13)

$$E_t = \frac{4}{P_y} \left( \frac{P}{P_y} - 1 \right) E \text{ for } P > 0.5 P_y$$  \hspace{1cm} (3.14)

The terms $\eta_A$ and $\eta_B$ are scalar parameters that allow for gradual inelastic stiffness reduction of the element associated with plastification at ends A and B, respectively. These terms are equal to 1.0 when the element is elastic, and zero when a plastic hinge is formed. The parameter $\eta$ is assumed to vary according to the parabolic function as

$$\eta = 1.0 \text{ for } \alpha \leq 0.5$$  \hspace{1cm} (3.15)

$$\eta = 4\alpha (1 - \alpha) \text{ for } 0.5 < \alpha \leq 1.0$$  \hspace{1cm} (3.16)

$$\eta = 0 \text{ for } \alpha > 1$$  \hspace{1cm} (3.17)

where $\alpha$ is a force-state parameter that measures the magnitude of axial force and bending moment of element. The term $\alpha$ can be expressed by AISC-LRFD or Orbison yield surfaces shown in Fig. 3.1 as

For AISC-LRFD plastic strength surface

$$\alpha = p + \frac{8}{9} m_y + \frac{8}{9} m_z \text{ for } p \geq \frac{2}{9} m_y + \frac{2}{9} m_z$$  \hspace{1cm} (3.18)

$$\alpha = \frac{p}{2} + m_y + m_z \text{ for } p < \frac{2}{9} m_y + \frac{2}{9} m_z$$  \hspace{1cm} (3.19)

For Orbison plastic strength surface (Orbison, McGuire and Abel, 1982)
\[ \alpha = 1.15 p^2 + m_z^2 + m_y^4 + 3.67 p^2 m_z^2 + 3.0 p^6 m_z^2 + 4.65 m_z^4 m_y^2 \]  
(3.20)

where \( P = P_y / P \), \( m_z = M_z / M_{pc} \) (strong-axis), \( m_y = M_y / M_{py} \) (weak-axis); \( P_y, M_{sp}, \) and \( M_{zp} \) are squash load, and plastic moment capacity of the cross-section about the y and z axes.

When the force point moves inside or along the initial yield surface \( (\alpha \leq 0.5) \), the element remains fully elastic without stiffness reduction. If the force point moves beyond the initial yield surface and inside the full yield surface \( (0.5 < \alpha \leq 1.0) \), the element stiffness is reduced to account for the effect of plastification at the element ends. The reduction of element stiffness is assumed to vary according to the parabolic function in Eq. (3.16). When member forces violate the plastic strength surface \( (\alpha > 1.0) \), the member forces will be scaled down to move the force point return the yield surface based on the incremental-iterative scheme.

(a) AISC-LRFD

(b) Orbison

Fig. 3.1 Full plastification surfaces
2.1.3 Shear Deformation Effect

To account for transverse shear deformation effect in a beam-column element, the stiffness coefficients of beam-column element should be modified. The flexibility matrix can be obtained by inverting the flexural stiffness matrix as

\[
\begin{align*}
\begin{bmatrix}
\theta_{MA} \\
\theta_{MB}
\end{bmatrix} &=
\begin{bmatrix}
\frac{k_{ii}}{k_{ii}k_{jj} - k_{ij}^2} & -\frac{k_{ij}}{k_{ii}k_{jj} - k_{ij}^2} \\
-k_{ij} & \frac{k_{ij}}{k_{ii}k_{jj} - k_{ij}^2}
\end{bmatrix}
\begin{bmatrix}
M_A \\
M_B
\end{bmatrix}
\end{align*}
\]  

(3.21)

where \(k_{ii}, k_{ij}\) and \(k_{ij}\) are components of stiffness matrix of planar beam-column; \(\theta_{MA}\) and \(\theta_{MB}\) are the slope of the neutral axis due to bending moment. The flexibility matrix corresponding to shear deformation can be written as

\[
\begin{align*}
\begin{bmatrix}
\theta_{SA} \\
\theta_{SB}
\end{bmatrix} &=
\begin{bmatrix}
\frac{1}{GA_sL} & \frac{1}{GA_sL} \\
\frac{1}{GA_sL} & \frac{1}{GA_sL}
\end{bmatrix}
\begin{bmatrix}
M_A \\
M_B
\end{bmatrix}
\end{align*}
\]  

(3.22)

where \(GA_s\) and \(L\) are shear rigidity and length of the beam-column element, respectively. The total rotations at the two ends A and B are obtained by combining Eqs. (3.21) and (3.22) as

\[
\begin{align*}
\begin{bmatrix}
\theta_A \\
\theta_B
\end{bmatrix} &=
\begin{bmatrix}
\theta_{MA} \\
\theta_{MB}
\end{bmatrix} +
\begin{bmatrix}
\theta_{SA} \\
\theta_{SB}
\end{bmatrix}
\end{align*}
\]  

(3.23)

The basic force and deformation relationship including shear deformation is derived by inverting the flexibility matrix as
\[
\begin{bmatrix}
M_A \\
M_B
\end{bmatrix} = \begin{bmatrix}
k_{ij}k_{yy} - k_{ij}^2 + k_{ij}A_{y}GL & -k_{ij}k_{yy} + k_{ij}^2 + k_{ij}A_{y}GL \\
k_{ii} + k_{ij} + 2k_{ij} + A_{y}GL & k_{ii} + k_{ij} + 2k_{ij} + A_{y}GL \\
-k_{ii}k_{ij} + k_{ij}^2 + k_{ij}A_{y}GL & k_{ii}k_{ij} - k_{ij}^2 + k_{ij}A_{y}GL \\
k_{ij} + k_{ij} + 2k_{ij} + A_{y}GL & k_{ij} + k_{ij} + 2k_{ij} + A_{y}GL
\end{bmatrix}\begin{bmatrix}
\theta_A \\
\theta_B
\end{bmatrix}
\] (3.24)

The member basic force and deformation relationship can be extended for three-dimensional beam-column element as

\[
\begin{bmatrix}
P \\
M_{yA} \\
M_{yB} \\
M_{zA} \\
M_{zB} \\
T
\end{bmatrix} = \begin{bmatrix}
\frac{E}{L}A & 0 & 0 & 0 & 0 & 0 \\
0 & C_{iyy} & C_{iyj} & 0 & 0 & 0 \\
0 & C_{ijy} & C_{ijj} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{icz} & C_{icz} & 0 \\
0 & 0 & 0 & C_{icz} & C_{icz} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{GJ}{L}
\end{bmatrix}\begin{bmatrix}
\delta \\
\theta_{yA} \\
\theta_{yB} \\
\theta_{zA} \\
\theta_{zB} \\
\phi
\end{bmatrix}
\] (3.25)

where

\[
C_{iyy} = \frac{k_{iy}k_{yy} - k_{iy}^2 + k_{iy}A_{y}GL}{k_{iy} + k_{ij} + 2k_{ij} + A_{y}GL}
\] (3.26)

\[
C_{ijy} = \frac{-k_{iy}k_{ij} + k_{ij}^2 + k_{ij}A_{y}GL}{k_{iy} + k_{ij} + 2k_{ij} + A_{y}GL}
\] (3.27)

\[
C_{ijy} = \frac{k_{iy}k_{ij} - k_{ij}^2 + k_{ij}A_{y}GL}{k_{iy} + k_{ij} + 2k_{ij} + A_{y}GL}
\] (3.28)

\[
C_{ijz} = \frac{k_{iz}k_{ij} - k_{ij}^2 + k_{ij}A_{z}GL}{k_{iz} + k_{ij} + 2k_{ij} + A_{z}GL}
\] (3.29)

\[
C_{ijz} = \frac{-k_{iz}k_{ij} + k_{ij}^2 + k_{ij}A_{z}GL}{k_{iz} + k_{ij} + 2k_{ij} + A_{z}GL}
\] (3.30)
\[
C_{jic} = \frac{k_{ijc} k_{ijc} - k_{ijc}^2 + k_{ijc} A_{sjy} GL}{k_{ic} + k_{ijc} + 2k_{ijc} + A_{sjy} GL}
\]  
(3.31)

where \( A_{sjy} \) and \( A_{sjc} \) are the shear areas with respect to \( y \) and \( z \) axes, respectively.

### 2.1.4 Element Stiffness Matrix accounting for \( P - \Delta \) Effect

In this section, the member basic force and deformation relationship in Eq. (3.25) is assembled into the structure stiffness matrix related to the nodal force and nodal displacement. The incremental end forces and displacements used in Eq. (3.25) are shown in Fig. 3.2a. The sign convention for the positive directions of elemental end forces and displacements of a beam-column element is shown in Fig. 3.2b. By comparing the two figures, the equilibrium and kinematic relationships can be expressed in symbolic form as

\[
\{f_n\} = [T]_{6x12}^T \{f_e\} 
\]  
(3.32)

\[
\{d_e\} = [T]_{6x12} \{d_L\} 
\]  
(3.33)

where \( \{f_n\} \) and \( \{d_e\} \) are the incremental nodal force and displacement vectors of a beam-column member expressed as

\[
\{f_n\} = \{r_{n1} \ r_{n2} \ r_{n3} \ r_{n4} \ r_{n5} \ r_{n6} \ r_{n7} \ r_{n8} \ r_{n9} \ r_{n10} \ r_{n11} \ r_{n12}\} 
\]  
(3.34)

\[
\{d_L\} = \{d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6 \ d_7 \ d_8 \ d_9 \ d_{10} \ d_{11} \ d_{12}\} 
\]  
(3.35)

and \( \{f_e\} \) and \( \{d_e\} \) are the incremental basic force and displacement vectors of a beam-column element in Eq. (3.25) expressed as

\[
\{f_e\} = \{P \ M_{yA} \ M_{yB} \ M_{zA} \ M_{zB} \ T\} 
\]  
(3.36)
\[
\{d_e\}^T = \{\delta, \theta_y^A, \theta_z^B, \theta_z^A, \theta_y^B, \phi\}
\]  
(3.37)

Fig. 3.2 Element end force and displacement notations

and \([T]_{6\times12}\) is a transformation matrix written as

\[
[T]_{6\times12} = 
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 1 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 0 & 1 \\
0 & 1/L & 0 & 0 & 0 & 1 & 0 & -1/L & 0 & 0 & 0 & 0 \\
0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
\end{bmatrix}
\]  
(3.38)

Using the transformation matrix by equilibrium and kinematic relations, the force-displacement relationship of a beam-column member may be written as

\[
\{f_r\} = [K_n]\{d_L\}
\]  
(3.39)

where \([K_n]\) is the element stiffness matrix expressed as

74
\[
[K_n]_{12 \times 12} = [T]^T [K_e]_{6 \times 6} [T]_{6 \times 12}
\] (3.40)

Eq. (3.40) can be sub-grouped as
\[
[K_n]_{12 \times 12} = \begin{bmatrix}
[K_n]_1 & [K_n]_2 \\
[K_n]^T_2 & [K_n]_3
\end{bmatrix}
\] (3.41)

where
\[
[K_n]_1 = \begin{bmatrix}
a & 0 & 0 & 0 & 0 \\
0 & b & 0 & 0 & 0 \\
0 & 0 & d & 0 & -e \\
0 & 0 & 0 & f & 0 \\
0 & 0 & -e & 0 & g \\
0 & c & 0 & 0 & 0 \\
\end{bmatrix}
\] (3.42)

\[
[K_n]_2 = \begin{bmatrix}
-a & 0 & 0 & 0 & 0 \\
0 & -b & 0 & 0 & 0 \\
0 & 0 & -d & 0 & -e \\
0 & 0 & 0 & -f & 0 \\
0 & 0 & e & 0 & i \\
0 & -c & 0 & 0 & 0 \\
\end{bmatrix}
\] (3.43)

\[
[K_n]_3 = \begin{bmatrix}
a & 0 & 0 & 0 & 0 \\
0 & b & 0 & 0 & 0 \\
0 & 0 & d & 0 & e \\
0 & 0 & 0 & f & 0 \\
0 & 0 & e & 0 & m \\
0 & c & 0 & 0 & 0 \\
\end{bmatrix}
\] (3.44)

\[
a = \frac{E_i A_i}{L}, \quad b = \frac{C_{ijc} + 2C_{jie} + C_{jij}}{L^2}, \quad c = \frac{C_{ijc} + C_{ijc}}{L} \\
d = \frac{C_{iiy} + 2C_{ijy} + C_{jij}}{L^2}, \quad e = \frac{C_{iiy} + C_{ijy}}{L}, \quad f = \frac{GJ}{L} \\
g = C_{iiy}, \quad h = C_{ijc}, \quad i = C_{ijy}, \quad j = C_{ijc}, \quad m = C_{ijy}, \quad n = C_{ijc}
\] (3.45)
Eq. (3.39) is used to enforce no side-sway in the member. If the member is permitted to sway, additional axial and shear forces will be induced in the member. These additional axial and shear forces due to member sway to the member end displacements can be related as

\[
\{ f_g \} = \begin{bmatrix} K_g \end{bmatrix} \{ d_L \} \quad (3.46)
\]

where

\[
\begin{bmatrix} K_g \end{bmatrix}_{12x12} = \begin{bmatrix} [K_s] & -[K_s] \\ -[K_s]^T & [K_s] \end{bmatrix} \quad (3.47)
\]

in which

\[
[K_s] = \begin{bmatrix} 0 & a & -b & 0 & 0 & 0 \\ a & c & 0 & 0 & 0 & 0 \\ -b & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.48)
\]

and

\[
a = \frac{M_{zA} + M_{zB}}{L^2}, \quad b = \frac{M_{yA} + M_{yB}}{L^2}, \quad c = \frac{P}{L} \quad (3.49)
\]

By combining Eqs. (3.39) and (3.46), the general force-displacement relationship of a beam-column element obtained as

\[
\{ f_L \} = [K_l] \{ d_L \} \quad (3.50)
\]

where

\[
\{ f_L \} = \{ f_n \} + \{ f_g \} \quad (3.51)
\]
\[
[K]_t = [K_n] + [K_e]
\]  
(3.52)

We obtain the stiffness matrix \([K]_t\) that includes the terms representing axial and flexural actions simultaneously.

### 2.2 Semi-rigid Connection Element

#### 2.2.1 Element Modeling

An independent zero-length element with three translational and three rotational springs was developed to simulate a general connection in 3D-framed analysis. The multi-spring element connects two nodes with coincident coordinates as shown in Fig. 3.3. In scope of this study, the translational and torsional springs have linear stiffness and fully rigid, while the two rotational ones (y and z axis) have linear or nonlinear stiffness. Coupling effects between the six springs of a connection are neglected.

![Connection element model](image)

Fig. 3.3 Space connection element model with zero-length

The relation between the incremental force vector \(\{F_s\}\) and displacement vector \(\{U_s\}\) of the multi-spring element corresponding to six degrees of freedom is as follows:

\[
\{F_s\} = [K_s]\{U_s\}
\]  
(3.53)
where \( \mathbf{K}_s \) is the diagonal tangent stiffness matrix for each multi-spring element, \( R_n^{\text{tra}} \) and \( R_n^{\text{rot}} \) are the component stiffness for the translational and rotational springs with respect to the \( n \) axis \((n = x, y, z)\).

2.2.2 Semi-Rigid Connection Models for Rotational Springs

In this study, the Kishi-Chen three-parameter power model (Kishi and Chen, 1987), the Richard-Abbott four-parameter model (Richard and Abbott, 1975), and the Chen-Lui exponential model (Lui and Chen, 1986) are used to evaluate the nonlinear behavior of semi-rigid connections. The independent hardening model is used to predict the cyclic behavior of the connections.

The Kishi-Chen model (Kishi and Chen, 1987) is currently one of the most popular models used for semi-rigid connections since it needs only three parameters to capture the moment-rotation curve and always gives a positive stiffness. The moment-rotation relationship of the connection is presented by Chen and Kishi as follows:

\[
M = \frac{R_{ki} |\theta_r|}{\left[1 + \left(\frac{|\theta_r|}{\theta_0}\right)^n\right]^{1/n}} \tag{3.55}
\]

where \( M \) and \( \theta_r \) are the moment and the rotation of the connection, \( n \) is the shape parameter, \( \theta_0 \) is the reference plastic rotation, and \( R_{ki} \) is the initial connection stiffness.
Richard and Abbott proposed a four-parameter model (Richard and Abbott, 1975). The moment-rotation relationship of the connection is defined by

\[
M = \frac{(R_{ki} - R_{kp})\theta_r}{1 + \left(\frac{(R_{ki} - R_{kp})\theta_r}{M_0}\right)^n} + R_{kp}\theta_r
\]

(3.56)

where \( M \) and \( \theta_r \) are the moment and the rotation of the connection, \( n \) is the parameter defining the shape, \( R_{ki} \) is the initial connection stiffness, \( R_{kp} \) is the strain-hardening stiffness and \( M_0 \) is the reference moment.

Lui and Chen (Lui and Chen, 1986) proposed the following exponential model:

\[
M = M_0 + \sum_{j=1}^{n} C_j \left(1 - \exp\left(-\frac{M}{2\alpha}\right)\right) + R_{sf}\theta_r
\]

(3.57)

in which \( M \) and \( |\theta_r| \) are the moment and the absolute value of the rotational deformation of the connection, \( \alpha \) is the scaling factor, \( R_{sf} \) is the strain-hardening stiffness of the connection, \( M_0 \) is the initial moment, \( C_j \) is the curve-fitting coefficient, and \( n \) is the number of terms considered.

In 1943, Ramberg and Osgood proposed a nonlinear stress-strain relationship model (Ramberg and Osgood, 1943). Ang and Morris standardized this model (Ang and Morris, 1984) as

\[
\frac{\theta}{\theta_o} = \left[\frac{M}{M_o}\left[1 + \left(\frac{M}{M_o}\right)^{n-1}\right]\right]^{n-1}
\]

(3.58)
where $M$ and $\theta$ are the moment and the rotation of the connection, $M_o$ and $\theta_o$ are the reference moment and the reference rotation, $n$ is the parameter defining the shape of the curve, and $R_{ki}$ is the initial connection stiffness.

2.2.3 Cyclic Behavior of Rotational Springs

![Diagram of the independent hardening model]

Fig. 3.4. The independent hardening model

The independent hardening model shown in Fig. 3.4 is used to represent for the cyclic behavior of semi-rigid connections because of its simple application (Chen and Saleeb, 1982). The virgin $M - \theta_r$ relationship is defined by the connection models in Eqs. (3.55-3.58). The instantaneous tangent stiffness of the connections is determined by taking derivative of Eqs. (3.55-3.58). Hysteretic behavior of semi-rigid connections is as follows:
1) If a connection is initially loaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows the line OA with the initial stiffness $R_{ki}$ shown in Fig. 3.4. The instantaneous tangent stiffness will be $R_{kt} = \frac{dM}{d[\theta_r]}$.

2) At point A, if the connection is unloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve goes back along the line ABC with the initial stiffness $R_{ki}$.

3) At point C, if the connection is continuously unloaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows the line CD with the initial stiffness $R_{ki}$ followed by the tangent stiffness $R_{kt}$.

4) At point D, if the connection is reloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve follows the straight line DE with the initial stiffness $R_{ki}$.

5) At point E, if the connection is continuously reloaded, the $M - \theta_r$ curve follows the line EF which is similar to the line OA.

6) At point F, the connection shows a similar curve to steps 1) - 5).

3. Nonlinear Solution Procedures

3.1 Nonlinear Static Algorithm

3.1.1 Formulation

Among several numerical methods, the Generalized Displacement Control (GDC) method proposed by Yang and Shieh (Yang and Shieh, 1990) appears to be one of the most robust and effective method for solving the nonlinear problems with multiple critical points as illustrated in Fig. 3.5 because of its general numerical stability and
efficiency. The incremental equilibrium equation of structure can be rewritten for the \( j \)th iteration of the \( i \)th incremental step as

\[
\begin{bmatrix}
K^i_{j-1}
\end{bmatrix}\{\Delta D^i_j\} = \lambda^i_j \{\hat{P}\} + \{R^i_{j-1}\}
\]  
(3.59)

where \( [K^i_{j-1}] \) is the tangent stiffness matrix, \( \{\Delta D^i_j\} \) is the displacement increment vector, \( \{\hat{P}\} \) is the reference load vector, \( \{R^i_{j-1}\} \) is the unbalanced force vector, and \( \lambda^i_j \) is the load increment parameter.

![Fig. 3.5 General characteristic of nonlinear systems](image)

According to (Batoz and Dhatt, 1979), Eq. (3.58) can be decomposed into the following equations:

\[
\begin{bmatrix}
K^i_{j-1}
\end{bmatrix}\{\Delta \hat{D}^i_j\} = \{\hat{P}\}
\]  
(3.60)

\[
\begin{bmatrix}
K^i_{j-1}
\end{bmatrix}\{\Delta \tilde{D}^i_j\} = \{R^i_{j-1}\}
\]  
(3.61)
\[
\{\Delta D_j^i\} = \lambda_j^i \{\Delta \hat{D}^i\} + \{\Delta D_j^i\}
\]  
(3.62)

Once the displacement increment vector \(\{\Delta D_j^i\}\) is determined, the total displacement vector \(\{D_j^i\}\) of the structure at the end of \(j\) th iteration can be accumulated as

\[
\{D_j^i\} = \{D_{j-1}^i\} + \{\Delta D_j^i\}
\]
(3.63)

The total applied load vector \(\{P_j^i\}\) at the \(j\) th iteration of the \(i\) th incremental step relates to the reference load vector \(\{\hat{P}\}\) as

\[
\{P_j^i\} = \Lambda_j^i \{\hat{P}\}
\]
(3.64)

where the load factor \(\Lambda_j^i\) can be related to the load increment parameter \(\lambda_j^i\) by

\[
\Lambda_j^i = \Lambda_{j-1}^i + \lambda_j^i
\]
(3.65)

The load increment parameter \(\lambda_j^i\) is an unknown. It is determined from a constraint condition. For the first iterative step \((j = 1)\), the load increment parameter \(\lambda_j^i\) is determined based on the Generalized Stiffness Parameter \((GSP)\) as

\[
\lambda_j^i = \lambda_1^i \sqrt{|GSP|}
\]
(3.66)

where \(\lambda_1^i\) is an initial value of load increment parameter, and the \(GSP\) is defined as

\[
GSP = \frac{\{\Delta \hat{D}^i\}^T \{\Delta \hat{D}^i\}}{\{\Delta \hat{D}^{i-1}\}^T \{\Delta \hat{D}^i\}}
\]
(3.67)

For the iterative step \((j \geq 2)\), the load increment parameter \(\lambda_j^i\) is calculated as
\[
\lambda^i_j = -\frac{\{\Delta \hat{D}^{i-1}_j\}^T \{\Delta \bar{D}^i_j\}}{\{\Delta \hat{D}^{i-1}_i\}^T \{\Delta \bar{D}^i_j\}}
\] 

(3.68)

where \(\{\Delta \hat{D}^{i-1}_i\}\) is the displacement increment generated by the reference load \(\hat{P}\) at the first iteration of the previous \((i-1)\) incremental step, and \(\{\Delta \hat{D}^i_j\}\) and \(\{\Delta \bar{D}^i_j\}\) denote the displacement increments generated by the reference load and unbalanced force vectors, respectively, at the \(j\) th iteration of the \(i\) th incremental step, as defined in Eqs. (3.60) and (3.61).

The major characteristics of the GSP defined in Eq. (3.67) are as follows:

1. The GSP is defined as the ratio of the norm displacement increments at the first incremental step to that of the current step. It represents the stiffness of the structure at the current incremental step with respect to the first incremental step. The GSP does not become unbounded the regions near the snap-back points, which is an advantage over the Current Stiffness Parameter (CSP) proposed by Bergan (Bergan et al., 1978; Bergan, 1980).

2. Since the sign of GSP depends on two vectors \(\{\Delta \hat{D}^{i-1}_i\}\) and \(\{\Delta \bar{D}^i_j\}\) as shown in Eq. (3.67) (plotted in Fig. 3.6), the GSP is always positive except for the incremental steps immediately after the limit points. Once a negative sign of GSP is detected, the load increment parameter \(\lambda^i_j\) computer from Eq. (3.66) should be multiplied by a minus sign to reverse the direction of loading when passing a limit point.
(3) The GSP begins with the value of unity and become zero at limit point. It increases as the structure is on the stage of stiffening, and decrease as the structure is on the stage of softening.

![Diagram of load and displacement](image)

**Fig. 3.6 Characteristics of GSP**

### 3.1.2 Application

The following lists the essential steps in the application of the generalized displacement control method:

**Step 1.** Select the initial load increment $\lambda_i^1$, the total number of incremental steps $N$, and the allowable load factor $\Lambda_{\text{max}}$.

**Step 2.** Initialize $S_{1n} = 4$, $S_{2n} = 2$, $\{P_0\} = \{0\}$, $\{R_0\} = \{0\}$, $\{D_0\} = \{0\}$, $\Lambda_0^i = \{0\}$.

**Step 3.** For the first iteration ($j = 1$) at each increment step $i$:

(a) Form the structure stiffness matrix $\begin{bmatrix} K_0^i \end{bmatrix}$.

(b) Solve for $\{\Delta D_i^i\}$ using Eq. (3.60).
(c) Calculate the load increment parameter $\lambda_i^j$: For $i = 1$, set $\lambda_i^j = \lambda_i^1$; for $i \geq 2$, use Eq. (3.67) to determine $GSP$ and calculate $\lambda_i^j$ using Eq. (3.66). Further, let $\lambda_i^j$ be of the same sign as $\lambda_i^{i-1}$. If $GSP$ is negative, multiply $\lambda_i^j$ by $-1$ to reverse the direction of loading.

Step 4. For the remaining iterations ($j \geq 2$):

(a) Update the structure stiffness matrix $[K_{j-1}']$: For structures of frame type, using the current axial force to modify the stability functions in Eqs. (3.2) and (3.3), and update the basic stiffness matrix in Eq. (3.25) to account for inelastic effects.

(b) Solve Eqs. (3.60) and (3.61) for $\{\Delta \hat{D}_j^i\}$ and $\{\Delta \bar{D}_j^i\}$, respectively.

(c) Determine the load increment parameter $\lambda_j^i$ using Eq. (3.68).

Step 5. Compute $\{\Delta D_j^i\}$ and $\Lambda_j^i$ using Eqs. (3.62) and (3.65), respectively.

Step 6. Update total displacement $\{D_j^i\}$ and external load $\{P_j^i\}$ using Eqs. (3.63) and (3.64), respectively.

Step 7. Calculate the internal force $\{F_j^i\}$ for the beam-column element:

(a) Extract the element end displacement $\{\Delta d_j^i\}$ from $\{\Delta D_j^i\}$

(b) Update the nodal coordinates and the transformation matrix

(c) Calculate the incremental basic displacement $\{\Delta d_e\}$ using Eq. (3.33).

(d) Calculate the incremental basic force $\{\Delta f_e\}$ using Eq. (3.25).

(e) Update the element member basic force using $\{f_j^i\}_e = \{f_{j-1}^i\}_e + \{\Delta f_e\}$.
(f) Check the yield condition: If the member basic force violates the yield surface, it is scaled down to return the force point to the yield surface.

(g) Form the element force in local coordinates using \( \{ f_j^i \}_i = \{ T \}^T \{ f_j^i \}_e \).

(h) Assemble the global internal force vector \( \{ F_j^i \} \).

Step 8. Calculate unbalanced force vector using \( \{ R_j^i \} = \{ P_j^i \} - \{ F_j^i \} \).

Step 9. Check the convergence: If the ratio of the norm of the unbalanced force \( \{ R_j^i \} \) to the norm of the applied force \( \{ P_j^i \} \) is smaller than a preset tolerance, go to step 10. Otherwise, let \( j = j + 1 \) and return to step 4.

Step 10. Check the termination: Whether the total number of steps is smaller than the preset number \( N \) or whether the load factor \( \Lambda_j^i \) is smaller than the allowable value \( \Lambda_{\text{max}} \), let \( i = i + 1 \) and go to step 3. Otherwise, stop the procedure.

A flow chart of the GDC algorithm is illustrated in Fig. 3.7.
BEGIN
Input data
Initialize
Form the stiffness matrix \([K'_{i,j}]\)
Solve for \(\Delta \hat{D}_j\) and \(\Delta \hat{D}_j\)
Determine \(\lambda_i\)
Calculate \(\Delta D'_j\)
Update \(D'_j\) and \(P'_j\)
Calculate the internal force \(F'_j\)
Calculate the unbalanced force \(R'_j\)
\(\|R'_j\|/\|P'_j\| < Tol\)
\(\lambda'_i < \lambda'_{max}\) or \(i < N\)
No

END
Next iteration \((j = j+1)\)
Next increment \((i = i+1)\)

Fig. 3.7 Flow chart of the GDC algorithm
3.2 Nonlinear Dynamic Algorithm

3.2.1 Formulation

An algorithm based on combination of the Hilber-Hughes-Taylor (HHT) method (Hilber, Hughes and Taylor, 1977) (also known as the alpha-method) and the Newton-Raphson method is proposed for the numerical integration of the nonlinear equation of motion because the HHT method possesses unconditional stability and second-order accuracy. In addition, it can induce numerical damping in the nonlinear solution which is not possible with the regular Newmark method (Newmark, 1959). The incremental equation of motion of a structure can be modified as

\[
[M] \{\Delta \ddot{v}^{t+\Delta t}\} + (1 + \alpha) [C_L] \{\Delta \dot{v}^{t+\Delta t}\} + (1 + \alpha) [K_T] \{\Delta v^{t+\Delta t}\} = \ldots
\]

\[
\{\Delta F_{ext}^{t+\Delta t}\} + \alpha [C_L] \{\Delta \dot{v}\} + \alpha [K_T] \{\Delta v\}
\]

where \(\{\Delta \ddot{v}\}\), \(\{\Delta \dot{v}\}\), and \(\{\Delta v\}\) are the vectors of incremental acceleration, velocity, and displacement, respectively; \([M]\), \([C_L]\), and \([K_T]\) are mass, damping, and tangent stiffness matrices, respectively; \(\{\Delta F_{ext}\}\) is the external incremental load vector; superscripts \(t\) and \(t+\Delta t\) are used to distinguish the values at time \(t\) and \(t+\Delta t\). The viscous damping matrix \([C_L]\) can be defined as Rayleigh damping matrix (Chopra, 2007):

\[
[C_L] = \alpha_M [M] + \beta_K [K_L]
\]

where \(\alpha_M\) and \(\beta_K\) are mass- and stiffness-proportional damping factors, respectively. In the scope of this paper, these coefficients are assumed to be constant in nonlinear dynamic time-history analysis. \([K_L]\) is the last-committed stiffness matrix. The last-committed stiffness matrix is the tangent stiffness matrix at the starting time of the new
time step, \([K_L]\) is unchanged during the unbalanced iterative procedure at each time step. If both modes are assumed to have the same damping ratio \(\xi\), then

\[
\alpha_M = \xi \frac{2\omega_1\omega_2}{\omega_1 + \omega_2}, \quad \beta_k = \xi \frac{2}{\omega_1 + \omega_2}
\]

(3.71)

where \(\omega_1\) and \(\omega_2\) are the natural frequencies of the first and second modes of the considered frame following decisive direction of dynamic loading.

Using Newmark’s approximate equations in standard form (Newmark, 1959) as:

\[
\{D^{i+\Delta t}\} = \{D^i\} + \Delta t \{\dot{D}^i\} + \left(\frac{1}{2} - \beta\right)\Delta t^2 \{\ddot{D}^i\} + \beta\Delta t^2 \{\dddot{D}^{i+\Delta t}\}
\]

(3.72)

\[
\{\dot{D}^{i+\Delta t}\} = \{\dot{D}^i\} + (1 - \gamma)\Delta t \{\dot{D}^i\} + \gamma\Delta t \{\ddot{D}^{i+\Delta t}\}
\]

(3.73)

Transforming Eqs. (3.72) and (3.73), the incremental velocity and acceleration at the first iteration of each time step can be written as

\[
\{\Delta \dot{D}^{i+\Delta t}\} = \frac{\gamma}{\beta\Delta t} \{\Delta D^{i+\Delta t}\} - \frac{\gamma}{\beta} \{\dot{D}^i\} + \left(1 - \frac{\gamma}{2\beta}\right)\Delta t \{\ddot{D}^i\}
\]

(3.74)

\[
\{\Delta \ddot{D}^{i+\Delta t}\} = \frac{1}{\beta\Delta t^2} \{\Delta D^{i+\Delta t}\} - \frac{1}{\beta\Delta t} \{\dot{D}^i\} - \frac{1}{2\beta} \{\ddot{D}^i\}
\]

(3.75)

Substituting Eqs. (3.74) and (3.75) into Eq. (3.69), the incremental displacement can be calculated from

\[
\left[\hat{K}\right]\{\Delta D^{i+\Delta t}\} = \{\Delta \hat{F}\}
\]

(3.76)

where \([\hat{K}]\) and \{\Delta \hat{F}\} are the effective stiffness matrix and incrementally effective force vector, respectively, given as
The residual forces in each time step can be eliminated by using the Newton-Raphson iterative procedure. At the first iteration of each time step, the total displacement, velocity, and acceleration at the time $t + \Delta t$ are updated based on the incremental displacement $\{\Delta D^{i+\Delta t}\}$ as follows:

$$\{D^{i+\Delta t}\} = \{D^i\} + \{\Delta D^{i+\Delta t}\}$$  \hspace{1cm} (3.79)

$$\{\dot{D}^{i+\Delta t}\} = \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\{\dot{D}^i\} + \left(1 - \frac{\gamma}{\beta}\right)\Delta t\{\ddot{D}^i\} + \frac{\gamma}{\beta.\Delta t}\{\Delta D^{i+\Delta t}\}$$  \hspace{1cm} (3.80)

$$\{\ddot{D}^{i+\Delta t}\} = \left(1 - \frac{1}{2\beta}\right)\{\ddot{D}^i\} - \frac{1}{\beta.\Delta t}\{\dot{D}^i\} + \frac{1}{\beta.\Delta t^2}\{\Delta D^{i+\Delta t}\}$$  \hspace{1cm} (3.81)

For the second and subsequent iterations of each time step, the structural system is solved under the effect of the residual force vector $\{R\}$ as

$$\left[\hat{K}\right]_k \{\delta D^{i+\Delta t}\}_{k+1} = \{R\}_k$$  \hspace{1cm} (3.82)

where the effective stiffness matrix $\left[\hat{K}\right]_k$ and the residual force vector $\{R\}_k$ are calculated at the unbalanced iterative step $k$, respectively, as follows

$$\left[\hat{K}\right]_k = (1 + \alpha)[K_T]_k + (1 + \alpha)\frac{\gamma}{\beta.\Delta t}[C_L]_k + \frac{1}{\beta.\Delta t^2}[M]$$  \hspace{1cm} (3.83)
\[
\{R\}_k = \left\{ F_{\text{ext}}^{i+\Delta t} \right\} - \left\{ F_{\text{int}} \right\}_k - \left\{ F_{\text{dam}} \right\}_k - \left\{ F_{\text{ine}} \right\}_k
\]

(3.84)

where \( \left\{ F_{\text{ext}}^{i+\Delta t} \right\} \) is the total external force vector; the inertial force vector \( \left\{ F_{\text{ine}} \right\}_k \), the damping force vector \( \left\{ F_{\text{dam}} \right\}_k \), and the updated internal force vector \( \left\{ F_{\text{int}} \right\}_k \) at the unbalanced iterative step \( k \) are respectively defined as:

\[
\left\{ F_{\text{ine}} \right\}_k = [M]\left\{ \dot{D}^{i+\Delta t} \right\}_k
\]

(3.85)

\[
\left\{ F_{\text{dam}} \right\}_k = [C_d]\left\{ \dot{D}^{i+\Delta t} \right\}_k
\]

(3.86)

\[
\left\{ F_{\text{int}} \right\}_k = \left\{ F_{\text{int}} \left( \left\{ D^{i+\Delta t} \right\}_k \right) \right\}
\]

(3.87)

Once the convergence criterion is satisfied, the structural response is updated for the next time step as

\[
\left\{ \Delta D^{i+\Delta t} \right\}_{k+1} = \left\{ \Delta D^{i+\Delta t} \right\}_k + \left\{ \delta \Delta D^{i+\Delta t} \right\}_{k+1}
\]

(3.88)

\[
\left\{ D^{i+\Delta t} \right\}_{k+1} = \left\{ D^{i+\Delta t} \right\}_k = \left\{ \dot{D}' \right\} + \left\{ \Delta D^{i+\Delta t} \right\}_{k+1}
\]

(3.89)

\[
\left\{ \dot{D}^{i+\Delta t} \right\}_{k+1} = \left\{ \dot{D}^{i+\Delta t} \right\}_k = \left( 1 - \frac{\gamma}{2\beta} \right) \Delta t \left\{ \dot{D}' \right\} + \left( 1 - \frac{\gamma}{\beta} \right) \left\{ \dot{D}' \right\} + \frac{\gamma}{\beta.\Delta t} \left\{ \Delta D^{i+\Delta t} \right\}_{k+1}
\]

(3.90)

\[
\left\{ \ddot{D}^{i+\Delta t} \right\}_{k+1} = \left\{ \ddot{D}^{i+\Delta t} \right\}_k = \left( 1 - \frac{1}{2\beta} \right) \Delta t \left\{ \ddot{D}' \right\} - \frac{1}{\beta.\Delta t^2} \left\{ \Delta D^{i+\Delta t} \right\}_{k+1}
\]

(3.91)

3.2.2 Application

As indicated in (Hughes, 2000), the HHT method will possess the unconditional stability and second-order accuracy when \( \alpha \in \left[ -\frac{1}{3}, 0 \right] \) and
\[ \gamma = \frac{1}{2} - \alpha \quad \beta = \frac{(1 - \alpha)^2}{4} \] (3.92)

With the smaller value of \( \alpha \), the more numerical damping is induced in the solution, it is necessary to obtain a convergent solution for some complicated nonlinear problems. Especially if the choice \( \alpha = 0 \) leads to the average acceleration method of the Newmark family (\( \gamma = 0.5, \beta = 0.25 \)) with zero numerical damping.

The details of procedure for the application of the Hilber-Hughes-Taylor method and the Newton-Raphson iteration method are as follows:

Step 1. Predictor phase
(a) Form the effective stiffness matrix \( \hat{K} \)
(b) Form the effective force vector \( \hat{F} \)
(c) Solve for \( \{\Delta D\} \) using Eq. (3.76).

Step 2. Corrector phase (force recovery)
(a) Update structural configuration and motion
(b) Update the member force
(c) Plasticity check: If the member force violates the yield surface, it will be scaled down to bring the force point back to the yield surface. Otherwise, go to the next step

Step 3. Convergence
(a) Calculate the unbalanced force
(b) Check convergence: If the convergence exists, update structural configuration and motion at the time $t + \Delta t$ and go to next time step. Otherwise, apply the unbalanced force on the structure system and go to step 1.

A flow chart of the above procedure is illustrated in Fig. 3.8.

![Flow chart of the proposed algorithm for dynamic analysis](image)

Fig. 3.8 Flow chart of the proposed algorithm for dynamic analysis

4. Numerical Examples and Discussions

4.1 Static Problems

4.1.1 Vogel 2-D Portal Steel Frame

Vogel (Vogel, 1985) presented this portal rigid frame as the European calibration frame for static inelastic analysis. The initial out-of-plumb straightness and the ECCS residual stress model were assumed for the frame and its members as shown in Fig. 3.9. The rigid beam-to-column connections were replaced by semi-rigid ones to study the
second-order inelastic behavior considering the connection rigidity by Chen and Kim (Chen and Kim, 1997) using the plastic-hinge method. The values of three parameters for the Kishi-Chen power model of these semi-rigid connections are:

\[ R_{ki} = 280,000 \text{kip} \cdot \text{in} / \text{rad} \], \( M_u = 1,250 \text{kip} \cdot \text{in} \), and \( n = 0.98 \). The results of the proposed program using Orbison yield surface and existing studies in predicting the second-order inelastic response of frames with rigid, semi-rigid, and hinged beam-to-column connections shown in Fig. 3.10, in which NASF is a nonlinear finite element program for second-order spread-of-plasticity analysis of semi-rigid planar steel frames (Nguyen, 2010). It can be seen that the nonlinear load – displacement curves agree well.

![Fig. 3.9. Vogel portal frame with semi-rigid connections](image)

4.1.2 Stelmack Experimental 2-D Two-Story Steel Frame

This one-bay two-story flexibly connected steel frame was tested by Stelmack (Stelmack, 1982) and is selected as a benchmark frame in the present study (Fig. 3.11). The A36 W5x16 hot-rolled profile was used for all frame members. The connections used in the frame were bolted top and seat angles connections of L4x4x1/2 made of A36
with bolt fasteners of A325 ¾-in. D, and its experimental moment-rotation relationship is shown in Fig. 3.12. Gravity loads was first applied at third points of the beam of the first floor, and then a lateral load was applied as the second loading sequence.

![Graph showing load-displacement response of Vogel portal frame](image)

Fig. 3.10. Load – displacement response of Vogel portal frame

(PZ: plastic-zone method, PH: plastic-hinge method)

The three parameters of the Kishi-Chen power model are determined by a curve-fitting as follows: \( R_{si} = 37,000 \text{kip}\cdot\text{in} / \text{rad} \), \( M_u = 225 \text{kip}\cdot\text{in} \), and \( n = 0.90 \). The moment-rotation relationship of the connection by the experiment and curve-fitting data result in a good agreement as shown in Fig. 3.12. The lateral load-displacement curves obtained by the proposed program using Orbison yield surface and experimental work compare well in Fig. 3.13. As a result, the proposed analysis is adequate in predicting the behavior and strength of semi-rigid connections.
Fig. 3.11. Stelmack experimental two-story frame

Fig. 3.12. Moment – rotation behavior of Stelmack two-story frame
4.1.3 Liew Experimental 2-D Portal Steel Frame

Liew et al (Liew et al., 1997) performed a series of tests on a variety of portal frames and their joints in order to provide test results for calibration of analysis and design methodology of semi-rigid steel frames. The SRF3 portal semi-rigid frame under non-proportional loading shown in Fig. 3.14 is used in this research to validate the reliability of the proposed program in predicting the nonlinear behavior of steel frames. The vertical load $P$ and horizontal load $H$ are first applied proportionally until reaching the values of $P = 612$ kN and $H = 29$ kN. The horizontal load $H$ is then increased until the frame collapses while the gravity loads are kept constant. Based on the test data on moment-rotation relations of column-to-base and beam-to-column joints (Liew, Yu, Ng and Shanmugam, 1997), two groups of three parameters of Kishi-Chen power model are obtained by curve-fitting technique for those joints (Table 3.1) and the comparison of
test and proposed curve-fitting curves is shown in Fig. 3.15. It can be seen in Fig. 3.16 that the initial curve, the ultimate load value and the ductility of test and proposed results correlates well.

![Diagram of Liew SRF3 portal frame](image)

B2: 256.0x146.4x6.4x10.9; $f_y = 345$ MPa
C2: 209.6x205.2x9.3x14.2; $f_y = 336$ MPa

**Fig. 3.14. Liew SRF3 portal frame**

<table>
<thead>
<tr>
<th>Connection</th>
<th>$M_u$ (kNm)</th>
<th>$R_{ki}$ (kNm/rad)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column-to-base</td>
<td>430</td>
<td>321,000</td>
<td>0.265</td>
</tr>
<tr>
<td>Beam-to-column</td>
<td>300</td>
<td>25,000</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 3.1 Curve-fitting connection parameters of Liew SRF3 portal frame**
Fig. 3.15. Moment – rotation relations of SRF3 portal frame

Fig. 3.16. Load – displacement response of Liew SRF3 portal frame
4.1.4 Orbison 3-D Six-Story Steel Frame

Fig. 3.17. Orbison six-story space frame with semi-rigid connections
Fig. 3.17 shows a six-story space rigid frame analyzed previously by Orbison et al. (Orbison, McGuire and Abel, 1982) using plastic hinge approach and recently by Chiorean (Chiorean, 2009) beam-column method considering the spread-of-plasticity along the member length. A36 steel with the yield stress of 250 MPa and Young’s modulus of 206,850 MPa are used for all members. Uniform floor load of 9.6 kN/m² is converted into equivalent concentrated loads on the top of the columns. Wind loads are simulated by point loads of 53.376 kN in the Y-direction at every beam-column joints. One proposed element per member is used to model this structure.

Aside from the rigid frame case, two more cases of the frames with linear and nonlinear semi-rigid connections are proposed and investigated by Chiorean (Chiorean, 2009). The bolted top and seat angles connections were assumed for all beam-to-column connections of the frame and their parameters using Kishi-Chen power model are as follows: (1) for W12x53 and W12x87 beams framing about the major-axis of columns:
the fixity factor $g = 0.86$, ultimate moment $M_u = 300 \text{kN} \cdot \text{m}$, and shape factor $n = 1.57$; (2) for W12x26 beams framing about the weak-axis of columns: the fixity factor $g = 0.86$, ultimate moment $M_u = 200 \text{kN} \cdot \text{m}$, and shape factor $n = 0.86$. From the above fixity factor value, the initial connection stiffness is calculated by the following formulation (Chiorean, 2009)

$$R_{xi} = \frac{4EI_0}{L} \frac{3g}{4(1-g)}$$ (3.93)

where $I_0$ is the moment of inertia of the beam cross-section.

The load – displacement curves at point A at the roof (Fig. 3.18) predicted by the proposed program using Orbison yield surface for rigid, linear semi-rigid, and nonlinear semi-rigid frames compare well with those of Chiorean (Chiorean, 2009). It can be concluded that the proposed program is reliable in predicting the nonlinear inelastic behavior of space steel semi-rigid frames.

### 4.2 Dynamic Problems

A computer program written in FORTRAN programming language is developed based on the above-mentioned algorithm. The flow chart of the proposed program for the application of the HHT method and the Newton-Raphson method is illustrated in Fig. 3.8. In this study, the integration coefficients of the HHT method applied for the proposed program are $\alpha = 0$, $\gamma = 0.5$ and $\beta = 0.25$. Three earthquake records of the El Centro, the Northridge, and the San Fernando as shown in Fig. 2.8 are used as ground excitation in dynamic analysis. Their peak ground accelerations and time steps are listed in Table 2.1. Several numerical examples are presented and discussed to verify the
accuracy and efficiency of the proposed program in predicting the nonlinear response of semi-rigid steel frame structures subjected to dynamic loadings. For verification purposes, the predictions obtained from the proposed program are compared with available results reported in the literature, and those generated by SAP2000. It should be noted that SAP2000 did not provide a semi-rigid element accounting for nonlinear effects of beam-to-column connection, whereas the proposed element can consider these effects.

4.2.1 Chan 2-D Two-Story Steel Frame

![Fig. 3.19. Chan 2-D two-story steel frame](image_url)

A single-bay two-story 2-D steel frame with flexible beam-to-column connections was studied by Chan and Chui (Chan and Chui, 2000). The geometry and loading of the frame are given in Fig. 3.19. All the frame members are W8x48 with Young’s modulus E of $205 \times 10^6$ kN/m$^2$ and the yielding stress of the steel material $\sigma_y$ of 235 MPa is considered. An initial geometric imperfection of column $\psi$ of 1/438 is considered. The
vertical static loads are applied on the frame to consider the second-order effects followed by the horizontal forces applied suddenly at each floor during 0.5 second, as shown in Fig. 3.19. The lumped masses of 5.1 and 10.2 Tons are modeled at the top of columns and the middle of the beams, respectively. A time step $\Delta t$ of 0.001 second is chosen and viscous damping of structure is ignored in the dynamic analysis. The four parameters of the Richard-Abbott model for semi-rigid connections are: $R_{k_i} = 23,000 kN \cdot m / rad$, $R_{kp} = 70 kN \cdot m / rad$, $M_o = 180 kN \cdot m$, and $n = 1.6$. Table 3.2 shows the peak displacements generated by the proposed program, Chan and Chui (Chan and Chui, 2000), and SAP2000. As shown in Fig. 3.20, it can be seen that the results of the elastic responses obtained by the proposed element are in good agreement with those predicted by Chan and Chui (Chan and Chui, 2000), and those results match well with those of SAP2000. In the case of the inelastic responses as shown in Fig. 3.21, it can be recognized that the main source of the differences is due to the modeling of material nonlinearity. The proposed program employs the refined plastic hinge method based on stability functions, whereas Chan and Chui’s research and SAP2000 use the lumped plastic hinge method. The moment-rotation curves at connection C are also plotted in Fig. 3.22.
Table 3.2 Peak displacements (mm) of 2-D two-story steel frame

<table>
<thead>
<tr>
<th>Frame type</th>
<th>Max/Min</th>
<th>Analysis type</th>
<th>Present</th>
<th>Chan and Chui</th>
<th>SAP2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>Max</td>
<td>Elastic</td>
<td>98.82</td>
<td>93.25</td>
<td>98.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>137.37</td>
<td>125.25</td>
<td>135.55</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-81.96</td>
<td>-77.01</td>
<td>-81.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-2.89</td>
<td>0.34</td>
<td>-6.90</td>
</tr>
<tr>
<td>Linear semi-rigid</td>
<td>Max</td>
<td>Elastic</td>
<td>120.45</td>
<td>116.38</td>
<td>121.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>159.47</td>
<td>151.42</td>
<td>161.02</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-109.22</td>
<td>-108.90</td>
<td>-112.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-22.18</td>
<td>-9.82</td>
<td>-16.84</td>
</tr>
<tr>
<td>Nonlinear semi-rigid</td>
<td>Max</td>
<td>Elastic</td>
<td>140.73</td>
<td>138.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>184.75</td>
<td>181.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-84.12</td>
<td>-82.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>4.62</td>
<td>16.02</td>
<td></td>
</tr>
</tbody>
</table>

(a) Rigid connections
Fig. 3.20. Second-order elastic responses of 2-D two-story frame
(a) Rigid connections

(b) Linear semi-rigid connections
Fig. 3.21. Second-order inelastic responses of 2-D two-story frame
(a) Elastic responses

(b) Inelastic responses

Fig. 3.22. Hysteresis loops at the connection C for various analyses of 2-D two-story frame
4.2.2 Vogel 2-D Six-Story Steel Frame

Vogel (Vogel, 1985) presented the two-bay six-story rigid frame for static inelastic analysis. Chui and Chan (Chui and Chan, 1996) built in the semi-rigid joints at beam ends to study the dynamic behavior involving the connection flexibility, as shown in Fig. 3.23. An initial geometric imperfection \( \psi \) of 1/450 was considered for the column members. Young’s modulus was \( 205 \times 10^6 \) kN/m\(^2\), and Poisson’s ratio was 0.3. The
curve fitted parameters of the Chen-Lui exponential model for a flush end plate connection were as follows: 

\[ R_{ki} = 12,340.198 \text{kN.m/rad}; \quad R_{ki} = 108.924 \text{kN.m/rad}; \quad M_{\alpha} = 0.0 \text{kN.m}; \quad \alpha = 0.00031783; \quad C_1 = -28.286; \quad C_2 = 573.189; \quad C_3 = -3,433.98; \quad C_4 = 8,511.3; \]
\[ C_5 = -9,362.567; \quad \text{and} \quad C_6 = 3,832.899 \quad \text{(unit of} \quad C_i \quad \text{is kN.m)} \quad \text{(Ostrander, 1970).} \]

The material was assumed to be elastic throughout the analysis, and the viscous damping was ignored. The static loads distributed on beams of 31.7 and 49.1 kN/m² were converted to lumped masses at the nodal points. Four elements per beam member and one element per column member were used by Chui and Chan (Chui and Chan, 1996).

The proposed analysis uses only one element per member for both the beam and column member. The rigid, linear semi-rigid, and nonlinear semi-rigid connections are investigated in the present analysis. The natural frequencies for the cases of fully rigid and linear semi-rigid connections compared well with those of Chui and Chan (Chui and Chan, 1996), as listed in Table 3.3.

**Table 3.3 Comparison of fundamental natural frequencies \( \omega \) (rad/s)**

<table>
<thead>
<tr>
<th>Case</th>
<th>Chui and Chan</th>
<th>Proposed</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid connections</td>
<td>2.410</td>
<td>2.407</td>
<td>-0.12</td>
</tr>
<tr>
<td>Linear semi-rigid connections</td>
<td>1.660</td>
<td>1.668</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The time-displacement responses corresponding to the dynamic forces with the different frequencies (\( \omega \)) of 1.00, 1.66, 2.41, and 3.30 (rad/s) of the present elastic dynamic analysis agree well with Chui and Chan (Chui and Chan, 1996), as shown in Figs. 29-32. Especially, resonances are observed for the linear semi-rigid and rigid joint types shown in Figs. 30-31 when the frequency of the dynamic force is equal to the fundamental natural frequency of the frame. However, the nonlinear semi-rigid frame
does not show the resonance due to energy dissipation induced by the hysteretic
damping of nonlinear connections, shown in Fig. 3.30. To illustrate the effect of the
hysteretic damping at semi-rigid connections, the frame subjected to suddenly applied
forces ($F_1(t) = 10.23$ kN and $F_2(t) = 20.44$ kN) during one second is investigated. The
displacement amplitude of the nonlinear semi-rigid frame obviously decreases while the
linear semi-rigid and rigid ones do not as shown in Fig. 3.28. It is concluded that the
hysteretic damping of nonlinear connections plays a significant role in the semi-rigid
frames.

To investigate the effect of the different connection models on the overall structural
response, the Richard-Abbott and Ramberg-Osgood models are also used. The
parameters of the Richard-Abbott model are: $R_{ki} = 12,336.86$ kN.m/rad; $R_{kp} =
112.97$ kN.m/rad; $M_0 = 96.03$ kN.m; $n = 1.6$ (Chui and Chan, 1996). The parameters of
the Ramberg-Osgood model are: $\theta_0 = 0.00609$ rad; $M_0 = 75.1293$ kN.m; $n = 5.5$ (Chui
and Chan, 1996). While the moment-rotation curves at connection C in Fig. 3.30 show a
little difference between these models, the time-displacement responses are identical, as
plotted in Fig. 3.29. Therefore, it can be concluded that the use of different models does
not significantly affect the behavior of frames.
Fig. 3.24. Time-displacement response by second-order elastic analysis ($\omega = 1.00 \text{ rad/s}$)

Fig. 3.25. Time-displacement response by second-order elastic analysis ($\omega = 1.66 \text{ rad/s}$)
Fig. 3.26. Time-displacement response by second-order elastic analysis ($\omega = 2.41$ rad/s)

Fig. 3.27. Time-displacement response by second-order elastic analysis ($\omega = 3.30$ rad/s)
Fig. 3.28. Time-displacement response by second-order elastic analysis under sudden load during 1s: $F_1(t) = 10.23\, \text{kN}, F_2(t) = 20.44\, \text{kN}$

Fig. 3.29. Comparing time-displacement response of the three models ($\omega = 1.66\, \text{rad/s}$)
Fig. 3.30. Comparing hysteresis loops of the three models at connection C ($\omega = 1.66$ rad/s)
Nonlinear dynamic responses of two-story space frames with various connection types (fully rigid, linear semi-rigid, and nonlinear semi-rigid connections) subjected to an impulse force of 100kN are studied. The member sizes and properties of the frame are shown in Fig. 3.31 (Chan and Chui, 2000). Young’s modulus of 205,000 MPa is used for all members. Static vertical loads of 36.9 and 46.1 kN are applied in order to consider the $P-\Delta$ and $P-\delta$ effects. These static loads are considered as lumped masses at nodes. The Chen-Lui exponential model is used for the flush end plate connection of semi-rigid joints. The parameters of the model are as follows: $R_{ki} = 12,340.198\text{kN.m/rad}$; $R_{ij} = 108.924\text{kN.m/rad}$; $M_o = 0.0\text{kN.m}$; $\alpha = 0.00031783$; $C_1 = -28.286$; $C_2 = 573.189$; $C_3 = -3,433.98$; $C_4 = 8,511.3$; $C_5 = -9,362.567$; and $C_6 = \ldots$
3,832.899 (unit of $C_i$ is kN.m) (Ostrander, 1970). The connection stiffness about the weak-axis of the sections assumes to be one fifth of the stiffness about the strong-axis. Two elements per beam member and one per column are used for this analysis. A time step $\Delta t$ of 0.005 s is chosen.

The time-displacement response at point A and the hysteresis loops of moment-rotation at joint C are shown in Figs. 37-39, respectively. The results of both rigid and linear semi-rigid connection cases compare well with those of the SAP2000 software. It is noted that the SAP2000 software could not analyze the nonlinear semi-rigid frame. The nonlinear semi-rigid frame has a larger displacement and a longer period, as shown in Fig. 3.32, because it has greater flexibility than the rigid one due to the presence of the semi-rigid connections. In the nonlinear semi-rigid connection case, the response shows a displacement shift due to permanent rotational deformation at connections. It
was found that the nonlinear connections dampened the deflection due to energy dissipation.

Fig. 3.33. Moment-rotation curve of nonlinear strong-axis spring at connection C

Fig. 3.34. Moment-rotation curve of nonlinear weak-axis spring at connection C
4.2.4 3-D Two -Story Steel Frame subjected to Earthquakes

![3-D two-story frame subjected to earthquakes](image)

Fig. 3.35. 3-D two-story frame subjected to earthquakes

The next example is a two-story 3-D steel frame as shown in Fig. 3.35. The nonlinear inelastic dynamic response of two-story 3-D steel frames with various connection types (fully rigid, linear semi-rigid, and nonlinear semi-rigid connections) subjected to two different earthquakes (El Centro and Northridge) shown in Fig. 2.8 and Table 2.1 is studied. This example also aims at verifying the accuracy of the beam-column element in predicting the nonlinear effects of 3-D framed structures. Sections of all members are W8\times31. Young’s modulus and Poisson ratio of material are 200 GPa and 0.3 respectively. A steel yield stress of 500 MPa was assumed for the nonlinear inelastic time-history analysis of the frame. The Chen-Lui exponential model was used for the flush end plate connection of semi-rigid joints. The parameters of the model are: $R_{ss} = 12,340.198\text{kN.m/\text{rad}}$; $R_{sy} = 108.924\text{kN.m/\text{rad}}$; $M_o = 0.0\text{kN.m}$; $\alpha = 0.00031783$; $C_1 = -28.286$; $C_2 = 573.189$; $C_3 = -3,433.98$; $C_4 = 8,511.3$; $C_5 = -9,362.567$; and $C_6 =
3,832.899 (unit of $C_i$ is kN.m) (Ostrander, 1970). The connection stiffness about the weak-axis of the sections was assumed to be one fifth of the stiffness about the strong-axis. The masses lumped at the framed nodes were assumed to be 50 Ns²/mm. The earthquake excitations were applied in the X-direction. The first two periods corresponding to the frame types are compared with those of SAP2000 listed in Table 3.4, these periods are used to predict viscous damping of structure with damping ratio $\xi$ of 0.05.

In the numerical modeling, only one element per member is used in the proposed program. Since the beam element provided by SAP2000 can not capture accurately the second-order effects if only one element per member is used, the member needs to be divided into many elements to capture accurately the second-order effects. In this study, six elements per member are automatically used in the modeling of the framed structure in SAP2000. Moreover, SAP2000 does not also include a beam-to-column connection element considering the nonlinearity of the moment-rotation relationship.

<table>
<thead>
<tr>
<th>Frame type</th>
<th>Mode</th>
<th>SAP2000</th>
<th>Present</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>1</td>
<td>0.9849</td>
<td>0.9849</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2949</td>
<td>0.2949</td>
<td>0.00</td>
</tr>
<tr>
<td>Semi-rigid</td>
<td>1</td>
<td>1.2128</td>
<td>1.2126</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3109</td>
<td>0.3109</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The displacement responses at the roof of the rigid and semi-rigid frames generated by the proposed program and SAP2000 are shown in Fig. 3.36 and Fig. 3.37 for the El Centro and Northridge earthquakes, respectively. It can be observed that the obtained results are identical with those of SAP2000 in all cases of nonlinear elastic analysis. In
the case of the nonlinear semi-rigid frame, the response shows the displacement drift due to permanent rotational deformation at connections as shown in Fig. 3.37b. In nonlinear inelastic analysis, there is a slight difference between the proposed results and those of SAP2000 because the steel yielding model and the structural viscous damping are estimated differently. In this numerical analysis, SAP2000 is applied to the lumped plastic hinge model following the FEMA 356 specification and the initial structural stiffness matrix is used to calculate the damping matrix, whereas the proposed program uses the refined plastic hinge model and the damping matrix of structure is recalculated at each time step based on the tangent stiffness matrix of the structure.

(a) Second-order elastic responses of rigid frame
(b) Second-order elastic responses of semi-rigid frame

(c) Second-order inelastic responses of rigid frame
Fig. 3.36. Nonlinear time-history responses of 3-D two-story frame under El Centro earthquake

(d) Second-order inelastic responses of semi-rigid frame

(a) Second-order elastic responses of rigid frame
(b) Second-order elastic responses of semi-rigid frame

(c) Second-order inelastic responses of rigid frame
4.2.5 Orbison 3-D Six-Story Steel Frame – A Case Study

An Orbison 3-D six-story steel frame with semi-rigid connections is plotted in Fig. 3.17. Static problem of this frame are solved by Chiorean (Chiorean, 2009) and Ngo-Huu et al. (Ngo-Huu et al., 2012). The present study investigated the nonlinear dynamic behavior of the frame subjected to the El Centro and San Fernando earthquakes, these earthquake records are shown in Fig. 2.8 and Table 2.1. The elastic modulus and shear modulus are 206,850 MPa and 79,293 MPa, respectively. A yield stress of 250 MPa is assumed for the nonlinear inelastic dynamic analysis of the frame. Three parameters of the Kishi-Chen model for the semi-rigid joints are listed in Table 3.5 (Chiorean, 2009). The lumped masses of 128.42 and 256.84 kN.sec$^2$/m are transferred from the uniform floor pressure of 9.6 kN/m$^2$, and they are assigned at the frame nodes as shown in Fig.
3.17b. The earthquake excitations are applied in the Y-direction. Viscous damping is considered by using the Rayleigh damping matrix with a damping ratio $\xi$ of 0.05, where the first two natural periods of the frames corresponding to the earthquake direction of Y are compared in Table 3.6. It can be seen that the obtained results by modal analysis of the proposed program and SAP2000 are nearly identical.

Fig. 3.38 and Fig. 3.39 show the lateral displacement responses at A-node appropriating to the rigid, linear semi-rigid, and nonlinear semi-rigid frames under the El Centro and San Fernando earthquakes, respectively. The results of peak displacement are compared in Table 3.7. It can be observed that a strong agreement of displacements predicted by the proposed program and SAP2000 is obtained in the second-order elastic analysis. Since using a different material-nonlinearity approach, the displacement responses of the proposed program and SAP 2000 have a small shift in the second-order inelastic analysis. Fig. 3.40 and Fig. 3.41 show hysteresis loops of the strong-axis rotational spring at the connection C corresponding to various earthquakes, respectively, which can not be obtained by SAP2000.

The peak displacement of the nonlinear semi-rigid frame is less than that of both the rigid and linear semi-rigid frames because of hysteretic damping of the nonlinear semi-rigid connections. It can be concluded that the beam-to-column connections play a significant role in the earthquake resistant design. The accurate prediction of real behavior of framed structures leads to both safer and more economic design.
Table 3.5 Parameters of semi-rigid connections follow the Kishi-Chen model

<table>
<thead>
<tr>
<th>Beam section</th>
<th>Bending-axis</th>
<th>$M_u$ (kN.m)</th>
<th>$R_{ki}$ (kN.m/rad)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12x87</td>
<td>Strong-axis</td>
<td>300</td>
<td>160,503.2</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>300</td>
<td>52,267.75</td>
<td>1.57</td>
</tr>
<tr>
<td>W12x53</td>
<td>Strong-axis</td>
<td>300</td>
<td>92,185.09</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>300</td>
<td>20,776.82</td>
<td>1.57</td>
</tr>
<tr>
<td>W12x26</td>
<td>Strong-axis</td>
<td>200</td>
<td>44,247.8</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>200</td>
<td>3,752.54</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 3.6 Comparison of first two natural periods (s) along applied earthquake direction of 3-D six-story steel frame

<table>
<thead>
<tr>
<th>Frame type</th>
<th>Mode</th>
<th>SAP2000</th>
<th>Present</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>1</td>
<td>5.5385</td>
<td>5.5386</td>
<td>+0.002</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.9958</td>
<td>1.9959</td>
<td>+0.005</td>
</tr>
<tr>
<td>Semi-rigid</td>
<td>1</td>
<td>6.0846</td>
<td>6.0797</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.1340</td>
<td>2.1338</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

Using the same personal computer configuration (AMD Phenom II X4 955 Processor, 3.2 GHz; 4.00 GB RAM), the analysis time of the proposed program and SAP2000 for the nonlinear inelastic time-history analysis of the six-story linear semi-rigid frame subjected to San Fernando earthquake, which is the problem having the longest analysis time among all cases, are 3 min 54 sec and 4 min 50 sec, respectively. This result demonstrates the high computational efficiency of the proposed program.
Table 3.7 Comparison of peak displacements (mm) at node A of 3-D six-story steel frame

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Max/min</th>
<th>Frame type - Analysis type</th>
<th>Displacement (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SAP2000</td>
<td>Present</td>
<td></td>
</tr>
<tr>
<td>El Centro</td>
<td>Max</td>
<td>RC - NE</td>
<td>248.19</td>
<td>249.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC - NI</td>
<td>220.17</td>
<td>222.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NE</td>
<td>338.22</td>
<td>342.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>238.07</td>
<td>237.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>251.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>248.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>RC - NE</td>
<td>-437.36</td>
<td>-437.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC - NI</td>
<td>-380.00</td>
<td>-368.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NE</td>
<td>-442.89</td>
<td>-442.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>-419.30</td>
<td>-428.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>-331.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>-375.80</td>
<td></td>
</tr>
<tr>
<td>San Fernando</td>
<td>Max</td>
<td>RC - NE</td>
<td>160.48</td>
<td>160.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC - NI</td>
<td>79.00</td>
<td>79.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NE</td>
<td>152.69</td>
<td>152.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>73.40</td>
<td>73.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>73.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>73.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>RC - NE</td>
<td>-162.30</td>
<td>-162.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC - NI</td>
<td>-216.41</td>
<td>-205.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NE</td>
<td>-125.38</td>
<td>-125.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>-203.02</td>
<td>-193.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>-78.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>-80.94</td>
<td></td>
</tr>
</tbody>
</table>

Note: Rigid Connections (RC), Linear semi-rigid Connections (LC), Nonlinear semi-rigid Connections (NC); Nonlinear Elastic analysis (NE), Nonlinear Inelastic analysis (NI)
(a) Second-order elastic responses of rigid frame

(b) Second-order elastic responses of semi-rigid frame
Fig. 3.38. Nonlinear time-history responses of 3-D six-story steel frame under Northridge earthquake
(a) Second-order elastic responses of rigid frame

(b) Second-order elastic responses of semi-rigid frame
(c) Second-order inelastic responses of rigid frame

(d) Second-order inelastic responses of semi-rigid frame

Fig. 3.39. Nonlinear time-history responses of 3-D six-story steel frame under San Fernando earthquake
(a) Second-order elastic analysis

(b) Second-order inelastic analysis

Fig. 3.40. Hysteresis loops of strong-axis rotational spring at connection C for various analyses of 3-D six-story steel frame subjected to El Centro earthquake
Fig. 3.41. Hysteresis loops of strong-axis rotational spring at connection C for various analyses of 3-D six-story steel frame subjected to San Fernando earthquake.
5. Summary and Conclusions

- A simple and effective numerical procedure for the nonlinear static and time-history analysis of three-dimensional steel frames considering geometry, material, and connection nonlinearities at the same time has been presented.
- A 3-D connection element including six degrees of freedom was developed and applied for beam-to-column joints.
- Nonlinear static equilibrium equations of structures are solved by adopting the General Displacement Control (GDC).
- An effective integration algorithm was developed for solving equations of motion of framed structures based on the combination of the Hilber-Hughes-Taylor method and the Newton Raphson method.
- Three main resources of damping of steel frames are taken into account in the proposed program are: (1) hysteretic damping due to inelastic material; (2) structural viscous damping employing Rayleigh damping; (3) hysteretic damping due to nonlinear beam-to-column connections.
- The proposed program was verified for accuracy and computational efficiency through several numerical examples with various static and dynamic loadings.
- It is also capable of accurately predicting natural periods of framed structures.
- The significant difference between the proposed program and SAP2000 is that SAP2000 cannot simulate nonlinear behavior of semi-rigid connections whereas the proposed one can, as illustrated in the Section 3.2.5.
- Moreover, by using only one element per member, the proposed program can accurately capture the second-order effects while the beam-column members of
SAP2000 need to be divided into several elements. This procedure saves computer resources, so it reduces computational time. Thus, it can be concluded that the proposed program can effectively be used for practical design in predicting nonlinear inelastic behavior of 3-D semi-rigid steel framed structures subjected to static and dynamic loadings instead of using the time consuming and costly commercial FEA software packages such as NASTRAN, ABAQUS, ANSYS, etc. which cannot accurately consider the nonlinear effect of semi-rigid connections by a simple and efficient manner.
Chapter 4. **SECOND-ORDER SPREAD-OF-PLASTICITY APPROACH FOR NONLINEAR STATIC AND DYNAMIC ANALYSIS OF THREE-DIMENSIONAL SEMI-RIGID STEEL FRAMES**

1. Introduction

In conventional analysis and design, beam-to-column connections are usually assumed to be fully rigid or ideally pinned joints. The real behavior of beam-to-column connections is a nonlinear curve which depends on the fabrication of connections. Such connections are called semi-rigid connections which play a role in transferring a part of moments from elements to other ones while the rest is resisted by themselves. The experimental studies showed that semi-rigid steel frames feature ductile and stable hysteretic behavior when the connections are designed appropriately (Azizinamini and Radziminski, 1989; Nader and Astaneh, 1991; Elnashai and Elghazouli, 1994; Elnashai, Elghazouli and Denesh-Ashtiani, 1998). The energy is dissipated through hysteresis loops of semi-rigid connections, which are one of the important damping sources of structures.

There are two common nonlinear analytical approaches for space steel framed structures: the plastic hinge methods (concentrated plasticity) (Hsieh and Deierlein, 1991; Gao and Haldar, 1995; Liew et al., 2000; Kim and Choi, 2001; Kim, Ngo-Huu and Lee, 2006; Ngo-Huu et al., 2007; Thai and Kim, 2009; Ngo-Huu, Nguyen and Kim, 2012) and the plastic zone methods (distributed plasticity) (Teh and Clarke, 1999; Jiang et al., 2002; Alemdar and White, 2005; Chiorean, 2009; Nguyen and Kim, 2014). The plastic hinge methods (Liew, Chen, Shanmugam and Chen, 2000; Kim and Choi, 2001;
Kim, Ngo-Huu and Lee, 2006; Ngo-Huu, Kim and Oh, 2007; Thai and Kim, 2009; Ngo-Huu, Nguyen and Kim, 2012) using stability functions obtained from the closed-form solution of the beam-column element subjected to end forces can accurately capture the second-order effects using only one or two elements per member. Material nonlinearity is considered by the lumped plastic hinges at the two ends of the member. The effects of distributed plasticity and residual stress are indirectly taken into account by using the reduced tangent modulus approach. However, the plastic hinge methods are limited due to their incapability of capturing the more complex member behaviors that involve torsional-flexural buckling, local buckling, and severe yielding under the combined action of compression and bi-axial bending, which may significantly reduce the load-carrying capacity of a structure (Jiang, Chen and Liew, 2002). Furthermore, the hinge methods have shown to over-estimate the limit strength when structural behavior is dominated by the instability of a few members (White and Chen, 1993). Also, it may inadequately give information as to what is happening inside the member because the member is assumed to remain fully elastic between its ends. In the meanwhile, the plastic zone methods based on interpolation shape functions requires members to be divided into several elements to accurately predict the second-order effects and spread-of-plasticity behavior of steel framed structures. It is generally recognized to be an “exact“ and computationally expensive solution compared with the plastic hinge methods.

In recent years, Alemdar and White (Alemdar and White, 2005) presented several beam-column finite element formulations for full nonlinear distributed plasticity analysis of planar frame structures. The fundamental processes within the derivation of
displacement-based, flexibility-based, and mixed element methods using Hermitian cubic polynomial functions are summarized. These formulations are presented using a total Lagrangian corotational approach and are also applicable to general beam-column elements for space structural analysis. Chiorean (Chiorean, 2009) proposed a beam-column method for nonlinear inelastic analysis of 3D semi-rigid steel frames. The nonlinear inelastic force-strain relationship and stability functions are used in representing the inelastic behavior and second-order effects, respectively. The advantage of this study is its ability to trace the spread of plasticity along the member length by using only one beam-column element per framed member in analysis modeling. However, it seems that the shape parameters $a$ and $n$ of the Ramberg-Osgood model and $\alpha$ and $p$ of the proposed modified Albermani model for the force-strain relationship of the cross-section, which considerably affect the inelastic behavior of the steel frames, are not consistently used. To overcome the limitations of the above mentioned studies, this study will develop a fiber beam-column element based on stability functions for nonlinear inelastic time-history analysis of space steel frames with semi-rigid connections.

This study presents a second-order spread-of-plasticity approach for nonlinear time-history analysis of space semi-rigid steel frames. The second-order effects are considered by the use of stability functions obtained from the closed-form solution of the beam-column element subjected to axial force and bending moments at the two ends. The spread of plasticity over the cross section and along the member length is captured by tracing uniaxial stress-strain relations of each fiber on the cross sections located at the selected integration points along the member length. A new force interpolation
function matrix is developed to consider the effects of moment magnification due to axial force and lateral displacements. Warping torsion and lateral-torsional buckling are ignored. An independent zero-length connection element with six translational and rotational springs is developed for beam-to-column joints with various connection types. This is efficient because modification of the beam-column stiffness matrix considering semi-rigid connections is unnecessary and the connection is ready to integrate with any element type. The Kishi-Chen three-parameter power model (Kishi and Chen, 1987) and the Richard-Abbott four-parameter model (Richard and Abbott, 1975) are applied for representing the moment-rotation relationship and predicting the instantaneous stiffness of connections. A numerical procedure based on the Hilber-Hughes-Taylor method combined with the Newton-Raphson method is developed to solve nonlinear differential equations of motion, while static equilibrium equation of structures is solved by the General Displacement Control algorithm. Several numerical examples are presented to verify the accuracy, efficiency, and applicability of the proposed program in predicting nonlinear inelastic static and dynamic responses of space steel frames with semi-rigid connections.

2. Nonlinear Finite Element Formulation

2.1 Second-Order Spread-of-Plasticity Beam-Column Element

2.1.1 The Effects of Small P-delta and Shear Deformation

To capture the effect of axial force acting on bending moment through the lateral displacement of the beam-column element (small P-delta effect), the stability functions reported by Chen and Lui (Chen and Lui, 1987) are used to minimize modeling and
solution time. Generally, only one element per member is needed to accurately capture the small P-delta effect. From Kim and Choi (Kim and Choi, 2001), the incremental force-displacement equation of space beam-column element accounting for transverse shear deformation effects can be expressed as

\[
\begin{align*}
\begin{bmatrix}
\Delta P \\
\Delta M_{yA} \\
\Delta M_{yB} \\
\Delta M_{zA} \\
\Delta M_{zB} \\
\Delta T
\end{bmatrix} &=
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & C_{1y} & C_{2y} & 0 & 0 & 0 \\
0 & C_{2y} & C_{1y} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{1z} & C_{2z} & 0 \\
0 & 0 & 0 & C_{2z} & C_{1z} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{GJ}{L}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \theta_{yA} \\
\Delta \theta_{yB} \\
\Delta \theta_{zA} \\
\Delta \theta_{zB} \\
\Delta \phi
\end{bmatrix}
\end{align*}
\]  
(4.1)

where \( \Delta P \), \( \Delta M_{yA} \), \( \Delta M_{yB} \), \( \Delta M_{zA} \), \( \Delta M_{zB} \), and \( \Delta T \) are the incremental axial force, end moments with respect to \( y \) and \( z \) axes, and torsion, respectively; \( \Delta \delta \), \( \Delta \theta_{yA} \), \( \Delta \theta_{yB} \), \( \Delta \theta_{zA} \), \( \Delta \theta_{zB} \), and \( \Delta \phi \) are the incremental axial displacement, joint rotations, and angle of twist, respectively; \( E \), \( G \) and \( J \) are the elastic modulus and shear modulus of a material and the torsional constant of a cross section respectively; \( C_{1y} \), \( C_{2y} \), \( C_{1z} \), and \( C_{2z} \) are bending stiffness coefficients accounting for the transverse shear deformation effects and are defined as

\[
C_{1y} = \frac{k_{1y}^2 - k_{2y}^2 + k_{1y}A_{zc}GL}{2k_{1y} + 2k_{2y} + A_{zc}GL}
\]  
(4.2)

\[
C_{2y} = \frac{-k_{1y}^2 + k_{2y}^2 + k_{2y}A_{zc}GL}{2k_{1y} + 2k_{2y} + A_{zc}GL}
\]  
(4.3)
\[ C_{1z} = \frac{k_{1z}^2 - k_{2z}^2 + k_{1z}A_y GL}{2k_{1z} + 2k_{2z} + A_y GL} \quad (4.4) \]

\[ C_{2z} = \frac{-k_{1z}^2 + k_{2z}^2 + k_{2y}A_y GL}{2k_{1z} + 2k_{2z} + A_y GL} \quad (4.5) \]

where \( k_{1n} = S_{1n} \left( EI_n / L \right) \) and \( k_{2n} = S_{2n} \left( EI_n / L \right) \); \( S_{1n} \) and \( S_{2n} \) are stability functions with respect to the axis of \( n \) \( (n = y, z) \) and are expressed as

\[
S_{1n} = \begin{cases} 
\frac{k_n L \sin (k_n L) - (k_n L)^2 \cos (k_n L)}{2 - 2 \cos (k_n L) - k_n L \sin (k_n L)} & \text{if } P < 0 \\
\frac{(k_n L)^2 \cosh (k_n L) - k_n L \sinh (k_n L)}{2 - 2 \cosh (k_n L) + k_n L \sinh (k_n L)} & \text{if } P > 0 
\end{cases} \quad (4.6) 
\]

\[
S_{2n} = \begin{cases} 
\frac{(k_n L)^2 - k_n L \sin (k_n L)}{2 - 2 \cos (k_n L) - k_n L \sin (k_n L)} & \text{if } P < 0 \\
\frac{k_n L \sin (k_n L) - (k_n L)^2}{2 - 2 \cosh (k_n L) + k_n L \sinh (k_n L)} & \text{if } P > 0 
\end{cases} \quad (4.7) 
\]

where \( k_n^2 = |P|/EI_n \). \( EA \) and \( EI_n \) denote the axial and bending stiffness of the beam-column element and are integrated as follows:

\[ EA = \sum_{j=1}^{s} w_j \left( \sum_{i=1}^{m} E_i A_i \right)_j \quad (4.8) \]

\[ EI_y = \sum_{j=1}^{s} w_j \left[ \sum_{i=1}^{m} E_i \left( A_y z_{ij}^2 + I_{yi} \right) \right]_j \quad (4.9) \]

\[ EI_z = \sum_{j=1}^{s} w_j \left[ \sum_{i=1}^{m} E_i \left( A_y y_{ij}^2 + I_{zi} \right) \right]_j \quad (4.10) \]
in which \( s \) is the total number of monitored sections along an element; \( m \) is the total number of fibers divided on the monitored cross-section; \( w_j \) is the weight coefficient for Lobatto quadrature at the \( j^{th} \) section (Michels, 1963); \( E_i \) and \( A_i \) are the elastic modulus of the material and the area of \( i^{th} \) fiber, respectively; \( I_{yi} \) and \( I_{zi} \) are the y-axis and z-axis moment of inertia of \( i^{th} \) fiber around its centroid; \( y_i \) and \( z_i \) are the coordinates of \( i^{th} \) fiber to the centroidal bending axis of the cross-section as shown in Fig. 4.1. The element force-deformation relationship of Eq. (4.1) can be expressed in symbolic form as

\[
\{\Delta F\} = [K_e]\{\Delta d\}
\]  

(4.11)
where

$$\{\Delta F\} = \begin{bmatrix} \Delta P & \Delta M_{yA} & \Delta M_{yB} & \Delta M_{zA} & \Delta M_{zB} & \Delta T \end{bmatrix}^T$$  \hspace{1cm} (4.12)

$$\{\Delta d\} = \begin{bmatrix} \Delta \delta & \Delta \theta_{yA} & \Delta \theta_{yB} & \Delta \theta_{zA} & \Delta \theta_{zB} & \Delta \phi \end{bmatrix}^T$$  \hspace{1cm} (4.13)

The element stiffness matrix is evaluated numerically by the Gauss-Lobatto integration rules (Michels, 1963) because this method allows for two integration points to coincide with the end sections of the element. Since inelastic behavior in beam elements often concentrates at the ends of the member, the monitoring of the end sections of the element is advantageous from the standpoint of accuracy and numerical stability. By contrast, the outermost integration points of other integration methods (e.g., the classical Gauss method, the Legendre-Gauss method, and the Newton-Cotes method, etc.) only approach the end sections with increasing order of integration but never coincide with the end sections, hence resulting in overestimation of the member strength (Spacone et al., 1996).

### 2.1.2 The Effect of Spread-of-Plasticity

In order to capture the spread of plasticity throughout the member length, a fiber beam-column model as shown in Fig. 4.1 is used. The fiber beam-column element is divided into a discrete number of monitored sections represented by the integration points. Each monitored section is divided into \(m\) small fibers, and each fiber is represented by its geometric characteristic, area \(A_i\), and its coordinate location corresponding to its centroid \((y_i, z_i)\). For hot-rolled steel sections, the residual stresses are assigned directly to fibers as the initial stresses. Fig. 4.2 illustrates the ECCS
residual stress pattern of the I-shape section used for this research (Kim, Ngo-Huu and Lee, 2006). Section deformations are represented by three strain resultants: the axial strain $\varepsilon$ and curvatures $\chi_z$ and $\chi_y$ with respect to $z$ and $y$ axes, respectively. The corresponding force resultants are the axial force $N$ and bending moments $M_z$ and $M_y$. The section forces and deformations are grouped in the following vectors:

Section force vector $\{Q\} = \begin{bmatrix} N & M_y & M_z \end{bmatrix}^T$ \quad (4.14)

Section deformation vector $\{q\} = \begin{bmatrix} \varepsilon & \chi_y & \chi_z \end{bmatrix}^T$ \quad (4.15)

![Diagram of ECCS residual stress pattern for I-section](image)

**Fig. 4.2 The ECCS residual stress pattern for I-section**

The incremental section force vector at each integration point is determined based on the incremental element force vector $\{\Delta F\}$ and the force interpolation function matrix $[B(x)]$ as

$$\{\Delta Q\} = [B(x)]\{\Delta F\}$$ \quad (4.16)
The incremental section deformation vector is determined based on the incremental section force vector and the section flexibility matrix as

$$\{\Delta q\} = [k_{sec}]^{-1}\{\Delta Q\}$$  \hspace{1cm} (4.18)

where $[k_{sec}]$ is the section stiffness matrix given as

$$[k_{sec}] = \begin{bmatrix}
\sum_{i=1}^{m} E_i A_i & \sum_{i=1}^{m} E_i A_i z_i & -\sum_{i=1}^{m} E_i A_i y_i \\
\sum_{i=1}^{m} E_i A_i z_i & \sum_{i=1}^{m} \left( A_i z_i^2 + I_{yi} \right) & -\sum_{i=1}^{m} E_i A_i y_i z_i \\
-\sum_{i=1}^{m} E_i A_i y_i & -\sum_{i=1}^{m} E_i A_i y_i z_i & \sum_{i=1}^{m} E_i \left( A_i y_i^2 + I_{zi} \right)
\end{bmatrix}$$  \hspace{1cm} (4.19)

Applying the assumption that plane sections remain plane after deformation and normal to the reference axis of the member (with the exception of warping deformations due to non-uniform torsion), the incremental longitudinal fiber strain vector is calculated from the incremental section deformation vector as

$$\{\Delta e\} = [l]\{\Delta q\}$$  \hspace{1cm} (4.20)

where $[l]$ is the linear geometric matrix given as follows

$$[l] = \begin{bmatrix}
1 & z_1 & -y_1 \\
1 & z_2 & -y_2 \\
\vdots & \vdots & \vdots \\
1 & z_m & -y_m
\end{bmatrix}$$  \hspace{1cm} (4.21)
Once the incremental fiber strain is evaluated, the incremental fiber stress is computed based on an appropriate stress-strain constitutive law for material. In this study, the elastic-perfectly plastic material model is applied for steel (neglecting the effect of strain hardening). The elastic modulus of each fiber $E_i$ will either be equal to the initial elastic modulus or be equal to zero for yielding fibers. By updating during the iteration process, axial and bending stiffness of each cross-section (Eq. (4.19)) obtained in this way are then integrated along the member length by Eqs. (4.8-4.10) to form the element tangent stiffness accounting for spread of plasticity.

The section resisting forces are computed by integrating the axial force and biaxial bending moment contributions of all fibers as

$$
\{Q_r\} = \begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} \sigma_i A_i \\ \sum_{i=1}^{m} \sigma_i A_i z_i \\ -\sum_{i=1}^{m} \sigma_i A_i y_i \end{bmatrix}
$$

(4.22)

where $\sigma_i$ is the normal stress of $i^{th}$ fiber accumulated from the incremental fiber stress itself. If the fiber is yielding, its normal stress will be assigned equal to the yield stress of steel $\sigma_i = \sigma_y$.

2.1.3 Element Stiffness Matrix accounting for the Effect of Large P-delta

The large P-delta effect is the influence of axial force $P$ acting through the relative transverse displacement of the member ends. This effect can be considered by using the geometric stiffness matrix $[K_g]$ as
\[
\begin{bmatrix}
K_g
\end{bmatrix}_{12\times12} = \begin{bmatrix}
[K_s] & -[K_s] \\
-[K_s]^T & [K_s]
\end{bmatrix}
\]
\hspace{1cm} (4.23)

where

\[
[K_s] = \begin{bmatrix}
0 & a & -b & 0 & 0 & 0 \\
a & c & 0 & 0 & 0 & 0 \\
-b & 0 & c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
\hspace{1cm} (4.24)

and

\[
a = \frac{M_{zA} + M_{zB}}{L^2}, \quad b = \frac{M_{yA} + M_{yB}}{L^2}, \quad c = \frac{P}{L}
\]
\hspace{1cm} (4.25)

The displacement of a beam-column element can be decomposed into two parts: the element deformation and rigid displacement. The incremental element deformation \(\{\Delta d\}\) in Eq. (4.13) can be obtained from the incremental element displacement \(\{\Delta D\}\) as

\[
\{\Delta d\} = [T]_{6\times12} \{\Delta D\}
\]
\hspace{1cm} (4.26)

where

\[
[T]_{6\times12} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 1 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 1 & 0 \\
0 & 1/L & 0 & 0 & 0 & 1 & 0 & -1/L & 0 & 0 & 0 & 0 \\
0 & 1/L & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0
\end{bmatrix}
\]
\hspace{1cm} (4.27)
The tangent stiffness matrix of a nonlinear beam-column element is obtained as follows:

$$ [K]_{12\times12} = [T]_{6\times12}^T [K_\tau]_{6\times6} [T]_{6\times12} + [K_\phi]_{12\times12} $$  \hspace{1cm} (4.28)

### 2.2 Nonlinear Beam-to-Column Connection Element

#### 2.2.1 Element Modeling

An independent zero-length multi-spring element with three translational and three rotational components is developed to simulate the connection of various elements. The multi-spring element can be used to connect two nodes with the same coordinate \((i \equiv j)\) as illustrated in Fig. 4.3. Since the purpose of this study is mainly to verify the bending moment transference in the major and minor axis of the beam-column element, the translational and torsional deformation components of the connection are prevented by modeling fully rigid springs in all numerical examples. The coupling effects between the six springs in a connection are also neglected.

The relation between the incremental force vector \(\{\Delta F_s\}\) and incremental deformation vector \(\{\Delta u_s\}\) of the multi-spring connection element is given by

$$ \{\Delta F_s\} = [K_{es}]{\Delta u_s} $$  \hspace{1cm} (4.29)

$$ [K_{es}] = \begin{bmatrix}
R_x^\delta & 0 & 0 & 0 & 0 & 0 \\
0 & R_y^\delta & 0 & 0 & 0 & 0 \\
0 & 0 & R_z^\delta & 0 & 0 & 0 \\
0 & 0 & 0 & R_y^\theta & 0 & 0 \\
0 & 0 & 0 & 0 & R_z^\theta & 0 \\
0 & 0 & 0 & 0 & 0 & R_x^\theta \\
\end{bmatrix} $$  \hspace{1cm} (4.30)
\[ \{\Delta F_s\} = \begin{bmatrix} \Delta P_x & \Delta P_y & \Delta P_z & \Delta M_y & \Delta M_z & \Delta M_x \end{bmatrix}^T \]  
\hfill (4.31)

\[ \{\Delta u_s\} = \begin{bmatrix} \Delta \delta_x & \Delta \delta_y & \Delta \delta_z & \Delta \theta_y & \Delta \theta_z & \Delta \theta_x \end{bmatrix}^T \]  
\hfill (4.32)

where \( K_{es} \) is the diagonal stiffness matrix of the connection element; \( R_n^\delta \) and \( R_n^\theta \) are the component stiffness of the translational and rotational springs with respect to the axis of \( n \) \((n = x, y, z)\); \( \Delta P_n, \Delta M_n, \Delta \delta_n, \) and \( \Delta \theta_n \) are the incremental spring forces, moments, axial deformations, and bending deformations of the connection element with respect to the axis of \( n \) \((n = x, y, z)\), respectively. The instantaneous tangent stiffness of the major and minor-axis rotational springs \( (R_z^\theta \text{ and } R_y^\theta) \) is given by

\[ R_n^\theta = \frac{dM}{d[\theta_r]} \]  
\hfill (4.33)

where the moment \( M \) is a nonlinear mathematical function following the relative rotation \( \theta_r \) at the connection represented by the Kishi-Chen three-parameter power model (Kishi and Chen, 1987) and the Richard-Abbott four-parameter model (Richard and Abbott, 1975).

Fig. 4.3 Modeling of space connection element with zero-length
The Kishi-Chen model (Kishi and Chen, 1987) is currently one of the most popular models used for semi-rigid connections since it needs only three parameters to capture the moment-rotation curve and always yields a positive stiffness. The moment-rotation relationship of the connection is presented by Chen and Kishi as follows:

\[
M = \frac{R_{ki} |\theta_r|}{1 + \left(\frac{|\theta_r|}{\theta_0}\right)^n} \quad (4.34)
\]

where \(M\) and \(\theta_r\) are the moment and the rotation of the connection, respectively, \(n\) is the shape parameter, \(\theta_0\) is the reference plastic rotation, and \(R_{ki}\) is the initial connection stiffness.

Richard and Abbott proposed a four-parameter model (Richard and Abbott, 1975). The moment-rotation relationship of the connection is defined by

\[
M = \frac{\left( R_{ki} - R_{kp} \right) |\theta_r|}{M_0 \left\{ 1 + \left( \frac{\left( R_{ki} - R_{kp} \right) |\theta_r|}{M_0} \right)^n \right\}} + R_{kp} |\theta_r| \quad (4.35)
\]

where \(M\) and \(\theta_r\) are the moment and the rotation of the connection, respectively, \(n\) is the shape parameter, \(R_{ki}\) is the initial connection stiffness, and \(R_{kp}\) is the strain-hardening stiffness and \(M_0\) is the reference moment.

Lui and Chen (Lui and Chen, 1986) proposed an exponential model as follows:

\[
M = M_0 + \sum_{j=1}^{n} C_j \left( 1 - \exp \left( \frac{|\theta_r|}{2\alpha} \right) \right) + R_{ef} |\theta_r| \quad (4.36)
\]
in which \( M \) and \( |\theta_r| \) are the moment and the absolute value of the rotational deformation of the connection, \( \alpha \) is the scaling factor, \( R_{xy} \) is the strain-hardening stiffness of the connection, \( M_0 \) is the initial moment, \( C_j \) is the curve-fitting coefficient, and \( n \) is the number of terms considered.

The incremental connection element deformation \( \{\Delta u_s\} \) in Eq. (4.32) can be obtained from the incremental connection element displacement \( \{\Delta U_s\} \) as

\[
\{\Delta u_s\} = [T_{s}]_{6\times12} \{\Delta U_s\}
\]  

(4.37)

where

\[
[T_{s}]_{6\times12} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(4.38)

The tangent stiffness matrix of a nonlinear connection element used for assembling the global structural stiffness matrix is obtained as follows:

\[
[K_s]_{12\times12} = [T_{s}]_{6\times12}^T [K_{s}]_{6\times6} [T_{s}]_{6\times12}
\]  

(4.39)
2.2.2 Cyclic Behavior of Rotational Springs

Fig. 4.4. The independent hardening model

The independent hardening model (Chen and Saleeb, 1982) shown in Fig. 4.4 is used to represent for the cyclic behavior of semi-rigid connections because of its simple application. The virgin $M - \theta_r$ relationship is defined by the connection models in Eqs. (4.34-4.36). The instantaneous tangent stiffness of the connections is determined by taking a derivative of Eqs. (4.34-4.36). Hysteretic behavior of semi-rigid connections is as follows:

1) If a connection is initially loaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows the line OA with the initial stiffness $R_{ki}$ shown in Fig. 4.4. The instantaneous tangent stiffness will be $R_{ki} = \frac{dM}{d[\theta_r]}$.

2) At point A, if the connection is unloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve goes back along the line ABC with the initial stiffness $R_{ki}$. 

155
3) At point C, if the connection is continuously unloaded, $M \cdot \Delta M$ is positive and the $M - \theta_r$ curve follows the line CD with the initial stiffness $R_{ki}$ followed by the tangent stiffness $R_{kt}$.

4) At point D, if the connection is reloaded, $M \cdot \Delta M$ is negative and the $M - \theta_r$ curve follows the straight line DE with the initial stiffness $R_{ki}$.

5) At point E, if the connection is continuously reloaded, the $M - \theta_r$ curve follows the line EF which is similar to the line OA.

6) At point F, the connection shows a similar curve to steps 1) - 5).

3. Nonlinear Solution Procedures

3.1 Nonlinear Static Algorithm

This section presents a numerical method for solving the static nonlinear equations of 3D framed structures. Among several numerical methods, the Generalized Displacement Control (GDC) method proposed by Yang and Shieh (Yang and Shieh, 1990) appears to be one of the most robust and effective methods for solving the static nonlinear problems with multiple critical points because of its general numerical stability and efficiency. The incremental form of equilibrium equation can be rewritten for the $j^{th}$ iteration of the $i^{th}$ incremental step as

$$
\left[ K_{j-1}^i \right] \{ \Delta D_j^i \} = \lambda_j^i \{ \hat{P} \} + \{ R_{j-1}^i \}
$$

(4.40)
where \([K'_{j-1}]\) is the tangent stiffness matrix of a structure, \(\{\Delta D'_j\}\) is the incremental displacement vector, \(\{\hat{P}\}\) is the reference load vector, \(\{R'_{j-1}\}\) is the unbalanced force vector, and \(\lambda'_j\) is the incremental load parameter.

Eq. (4.39) can be decomposed into the following equations

\[
\begin{align*}
[K'_{j-1}]\{\Delta D'_j\} &= \{\hat{P}\} \\
[K'_{j-1}]\{\Delta \bar{D}'_j\} &= \{R'_{j-1}\}
\end{align*}
\]

\[(4.41)\]  \[(4.42)\]

\[
\{\Delta D'_j\} = \lambda'_j \{\Delta \hat{D}'_j\} + \{\Delta \bar{D}'_j\}
\]

\[(4.43)\]

The incremental load parameter \(\lambda'_j\) is an unknown. It is determined from a constraint condition. For the first iterative step \((j = 1)\), the incremental load parameter \(\lambda'_j\) is determined based on the Generalized Stiffness Parameter \((GSP)\) as

\[
\lambda'_j = \lambda'^{1}_1 \sqrt{GSP}
\]

\[(4.44)\]

where \(\lambda'^{1}_1\) is an initial value of incremental load parameter, and the \(GSP\) is defined as

\[
GSP = \frac{\{\Delta \hat{D}'_1\}^T \{\Delta \hat{D}'_1\}}{\{\Delta \hat{D}'_{1-1}\}^T \{\Delta \hat{D}'_{1-1}\}}
\]

\[(4.45)\]

For the following iteration \((j \geq 2)\), the incremental load parameter \(\lambda'_j\) is given by

\[
\lambda'_j = -\frac{\{\Delta \hat{D}'_{j-1}\}^T \{\Delta \bar{D}'_j\}}{\{\Delta \hat{D}'_{j-1}\}^T \{\Delta \bar{D}'_{j-1}\} + \{\Delta \bar{D}'_j\}}
\]

\[(4.46)\]
where \(\{\Delta \hat{D}_i^{(i-1)}\}\) is the incremental displacement generated by the reference load \(\{\hat{P}\}\) at the first iteration of the previous incremental step \((i-1)\), and \(\{\Delta \hat{D}_j^i\}\) and \(\{\Delta \hat{D}_j^i\}\) denote the incremental displacements generated by the reference load and unbalanced force vectors, respectively, at the \(j^{th}\) iteration of the \(i^{th}\) incremental step, as defined in Eqs. (4.41) and (4.42).

The following is a step-by-step summary of solution algorithm focused on the element state determination process of a single iteration.

Step 1. Solve the global structure equilibrium equation and update the incremental beam-column element and connection element displacements, \(\{\Delta D\}\) and \(\{\Delta U,\}\), respectively.

Step 2. Compute the incremental element deformation, \(\{\Delta d\}\) and \(\{\Delta u,\}\), using Eq. (4.26) and Eq. (4.37).

Step 3. Compute the incremental element force, \(\{\Delta F\}\) and \(\{\Delta F,\}\), using Eq. (4.11) and Eq. (4.29) based on the element stiffness matrix, \([K_e]_{6x6}\) and \([K_{es}]_{6x6}\) of the previous step, respectively.

Step 4. Compute the incremental section force \(\{\Delta Q\}\) using Eq. (4.16).

Step 5. Compute the section stiffness \(\{k_{sec}\}\) using Eq. (4.19).

Step 6. Compute the incremental section deformation \(\{\Delta q\}\) using Eq. (4.18).

Step 7. Compute the incremental fiber strain \(\{\Delta e\}\) using Eq. (4.20) and update fiber strain \(\{e\}\).
Step 8. Compute the incremental fiber stress \( \Delta \sigma \) using \( \Delta \sigma = E_i \Delta e \) and update fiber stress \( \sigma \). If \( \sigma_i > \sigma_y \) assign \( \sigma_i = \sigma_y \) for the elastic-perfectly plastic material model of steel.

Step 9. If any fiber is in yielding state \( \sigma_i = \sigma_y \) assign the fiber elastic modulus to be equal to zero, \( E_i = 0 \).

Step 10. Compute the section resisting force \( \{Q\} \) using Eq. (4.22).

Step 11. Update stiffness of beam-column and connection elements, \([K_e]_{6\times6}\) and \([K_c]_{6\times6}\). Update tangent stiffness of beam-column and connection elements, \([K]_{12\times12}\) and \([K_s]_{12\times12}\).

Step 12. Assemble the structure resisting force and structure stiffness matrix.

Step 13. Compute structure unbalanced forces.

Step 14. Check for the structure convergence: If the structure unbalanced forces satisfy the specified tolerance (i.e., convergence is achieved), go to the next incremental load step. Otherwise, return to step 1 for the next iteration to eliminate the structure unbalanced forces.

3.2 Nonlinear Dynamic Algorithm

A nonlinear algorithm based on the Hilber-Hughes-Taylor (HHT) method (Hilber, Hughes and Taylor, 1977) (also known as the alpha method) is developed for solving governing differential equations of motion because the HHT method possesses unconditional numerical stability and second-order accuracy. In addition, it can induce numerical damping in the nonlinear solution which is impossible with the Newmark-
beta method (Newmark, 1959). The incremental equation of motion of a structure can be modified as

\[
\begin{align*}
&M \{\ddot{\Delta D}^{t+\Delta t}\} + (1 + \alpha)[C] \{\Delta \dot{D}^{t+\Delta t}\} + (1 + \alpha)[K_T] \{\Delta D^{t+\Delta t}\} = \\
&\{\Delta F^{t+\Delta t}_{ext}\} + \alpha[C]\{\Delta \dot{D}'\} + \alpha[K_T]\{\Delta D'\}
\end{align*}
\]

where the dissipation coefficient of \( \alpha \in \left[-\frac{1}{3}, 0\right] \) for accuracy and numerical stability; \( \{\Delta D\} \), \( \{\Delta \dot{D}\} \), and \( \{\Delta D\} \) are the vectors of incremental acceleration, velocity, and displacement, respectively; \( [M] \), \( [C] \), and \( [K_T] \) are mass, damping, and tangent stiffness matrices, respectively; \( \{\Delta F^{t+\Delta t}_{ext}\} \) is the external incremental load vector; and, superscripts \( t \) and \( t+\Delta t \) are used to distinguish the values at time \( t \) and \( t+\Delta t \). The structural viscous damping matrix \( [C] \) can be defined as Rayleigh damping (Chopra, 2007):

\[
[C] = \alpha_M [M] + \beta_K [K]
\]

where \( \alpha_M \) and \( \beta_K \) are the coefficients of mass- and stiffness-proportional damping, respectively. If both modes are assumed to have the same damping ratio \( \xi \), then

\[
\alpha_M = \xi \frac{2 \omega_1 \omega_2}{\omega_1 + \omega_2} \quad ; \quad \beta_K = \xi \frac{2}{\omega_1 + \omega_2}
\]

where \( \omega_1 \) and \( \omega_2 \) are the natural frequencies of the first and second modes of the frame, respectively.
Using Newmark’s approximate equations in standard form as shown in (Newmark, 1959) and using coefficients $\gamma = \frac{1}{2} - \alpha$ and $\beta = \frac{(1 - \alpha)^2}{4}$ proposed by Hughes (Hughes, 2000), we have:

\[
\begin{align*}
\{D^{+\Delta t}\} &= \{D^t\} + \Delta t \{\dot{D}^t\} + \left(\frac{1}{2} - \beta\right) \Delta t^2 \{\ddot{D}^t\} + \beta \Delta t^2 \{\dddot{D}^{+\Delta t}\} \tag{4.50} \\
\{\dot{D}^{+\Delta t}\} &= \{\dot{D}^t\} + (1 - \gamma) \Delta t \{\ddot{D}^t\} + \gamma \Delta t \{\dddot{D}^{+\Delta t}\} \tag{4.51}
\end{align*}
\]

Transforming Eqs. (4.50) and (4.51), the incremental velocity and acceleration vectors at the first iteration of each time step can be written as

\[
\begin{align*}
\{\Delta \dot{D}^{+\Delta t}\} &= \frac{\gamma}{\beta \Delta t} \{\Delta D^{+\Delta t}\} - \frac{\gamma}{\beta} \{\dot{D}^t\} + \left(1 - \frac{\gamma}{2 \beta}\right) \Delta t \{\ddot{D}^t\} \tag{4.52} \\
\{\Delta \ddot{D}^{+\Delta t}\} &= \frac{1}{\beta \Delta t^2} \{\Delta D^{+\Delta t}\} - \frac{1}{\beta \Delta t} \{\dot{D}^t\} - \frac{1}{2 \beta} \{\ddot{D}^t\} \tag{4.53}
\end{align*}
\]

Substituting Eqs. (4.52) and (4.53) into Eq. (4.47), the incremental displacement vector can be calculated from

\[
\begin{align*}
\hat{K} \{\Delta D^{+\Delta t}\} = \{\Delta \hat{F}\} \tag{4.54}
\end{align*}
\]

where $\hat{K}$ and $\{\Delta \hat{F}\}$ are the effective stiffness matrix and incremental effective force vector, respectively, given as

\[
\begin{align*}
\hat{K} &= (1 + \alpha) [K_r] + (1 + \alpha) \frac{\gamma}{\beta \Delta t} [C] + \frac{1}{\beta \Delta t^2} [M] \tag{4.55}
\end{align*}
\]
\[
\{\Delta \dot{F}\} = \{\Delta F^{t+\Delta t}\} + \alpha [K_T]\{\Delta D^t\} + [M] \left\{\frac{1}{\beta_1 \Delta t}\{\dot{D}^t\} + \frac{1}{2 \beta_1}\{\ddot{D}^t\}\right\} + \\
[C] \left\{(1+\alpha)\frac{\gamma}{\beta_2}\{\dot{D}^t\} - (1+\alpha)\left(1 - \frac{\gamma}{2 \beta_2}\right)\Delta t\{\dot{D}^t\} + \alpha\{\Delta \dot{D}^t\}\right\}
\]

(4.56)

Unbalanced forces in each time step can be eliminated by using the well-known Newton-Raphson iterative method. At the first iteration of each time step, the total displacement, velocity and acceleration at the time \(t + \Delta t\) are updated based on the incremental displacement \(\{\Delta D^{t+\Delta t}\}\) as follows:

\[
\{D^{t+\Delta t}\} = \{D^t\} + \{\Delta D^{t+\Delta t}\}
\]

(4.57)

\[
\{\dot{D}^{t+\Delta t}\} = \left(1 - \frac{\gamma}{2 \beta_1}\right)\Delta t\{\dot{D}^t\} + \left(1 - \frac{\gamma}{\beta_1}\right)\{\dot{D}^t\} + \frac{\gamma}{\beta_1 \Delta t}\{\Delta D^{t+\Delta t}\}
\]

(4.58)

\[
\{\ddot{D}^{t+\Delta t}\} = \left(1 - \frac{1}{2 \beta_1}\right)\{\ddot{D}^t\} - \frac{1}{\beta_1 \Delta t}\{\dot{D}^t\} + \frac{1}{\beta_1 \Delta t^2}\{\Delta D^{t+\Delta t}\}
\]

(4.59)

For the second and subsequent iterations of each time step, the structural system is solved under the effect of the unbalanced force vector \(\{R\}\) as

\[
\left[\hat{K}\right]_k \{\delta \Delta D^{t+\Delta t}\}_{k+1} = \{R\}_k
\]

(4.60)

where the effective stiffness matrix \(\left[\hat{K}\right]_k\) and the residual force vector \(\{R\}_k\) are calculated at the unbalanced iterative step \(k\), respectively, as follows:

\[
\left[\hat{K}\right]_k = \left(1 + \alpha\right)[K_T]_k + \left(1 + \alpha\right)\frac{\gamma}{\beta_1 \Delta t}[C] + \frac{1}{\beta_1 \Delta t^2}[M]
\]

(4.61)

\[
\{R\}_k = \{F^{t+\Delta t}_{ext}\} - \{F_{int}\}_k - \{F_{dam}\}_k - \{F_{ine}\}_k
\]

(4.62)
where \( \{ F_{\text{ext}}^{\Delta t} \} \) is the total external force vector. The inertial force vector \( \{ F_{\text{in}} \} \), the damping force vector \( \{ F_{\text{damp}} \} \), and the updated internal force vector \( \{ F_{\text{int}} \} \) at the unbalanced iterative step \( k \) are respectively defined as:

\[
\begin{align*}
\{ F_{\text{in}} \}_k &= [M] \{ \dot{D}^{\Delta t} \}_k \\
\{ F_{\text{damp}} \}_k &= [C] \{ \dot{D}^{\Delta t} \}_k \\
\{ F_{\text{int}} \}_k &= \{ F_{\text{int}} \{ \{ D^{\Delta t} \}_k \} \}
\end{align*}
\]  

(4.63)

(4.64)

(4.65)

At each iterative step, the state of each fiber and characteristic of a cross section of each beam-column element, stiffness of each multi-spring connection element are updated for assembling the new structural stiffness matrix. Once the convergence criterion is satisfied, the structural response history is saved for the next time step as

\[
\begin{align*}
\{ \Delta D^{\Delta t+\Delta t} \}_{k+1} &= \{ \Delta D^{\Delta t} \}_k + \{ \delta \Delta D^{\Delta t} \}_{k+1} \\
\{ D^{\Delta t+\Delta t} \} &= \{ D^{\Delta t} \}_{k+1} = \{ \dot{D}' \} + \{ \Delta D^{\Delta t+\Delta t} \}_{k+1} \\
\{ \ddot{D}^{\Delta t+\Delta t} \} &= \{ \ddot{D}^{\Delta t} \}_{k+1} = \left( 1 - \frac{\gamma}{2\beta} \right) \Delta t \{ \ddot{D}' \} + \left( 1 - \frac{\gamma}{\beta} \right) \{ \ddot{D}' \} + \frac{\gamma}{\beta \Delta t} \{ \Delta D^{\Delta t+\Delta t} \}_{k+1} \\
\{ \dddot{D}^{\Delta t+\Delta t} \} &= \{ \dddot{D}^{\Delta t} \}_{k+1} = \left( 1 - \frac{1}{2\beta} \right) \{ \dddot{D}' \} - \frac{1}{\beta \Delta t} \{ \dddot{D}' \} + \frac{1}{\beta \Delta t^2} \{ \Delta D^{\Delta t+\Delta t} \}_{k+1}
\end{align*}
\]  

(4.66)

(4.67)

(4.68)

(4.69)
4. Numerical Examples and Discussions

4.1 Static Problems

4.1.1 Vogel Portal Steel Frame

Vogel (Vogel, 1985) presented the portal rigid frame as the European calibration frame for second-order inelastic analysis using the plastic zone method. The initial out-of-plumb straightness of $\psi = 1/400$ and the initial ECCS residual stress were assumed for the frame and its members (Fig. 4.5). Young’s modulus and the yield stress of steel are $E = 205 \text{kN/mm}^2$ and $\sigma_y = 235 \text{N/mm}^2$. For plastic zone analysis, Vogel used fifteen elements for the columns and fourteen elements for the beam, while the proposed program used only two integration points along the member length corresponding to two end sections. The cross section is discretized into sixty-six fibers (twenty four at both flanges, eighteen at the web). The main aim of this analysis is to demonstrate the capability of the proposed beam-column element in capturing the effects of both geometric and material nonlinearities accurately. The analysis results of the frame
obtained by using the proposed beam-column element are compared with those predicted by Vogel as illustrated in Fig. 4.6. It can be seen that the load-displacement curves without shear deformation of Vogel and the proposed program are identical.

The rigid beam-to-column connections were replaced by semi-rigid ones to study the second-order inelastic behavior including the connection nonlinearity by Chen and Kim (Chen and Kim, 1997) using the plastic-hinge method. The three parameters for Kishi-Chen power model of these semi-rigid connections are: $R_u = 31,635 \text{kN} \cdot \text{m} / \text{rad}$, $M_u = 142 \text{kN} \cdot \text{m}$, and $n = 0.98$. The results of the proposed program and NASF’s one in predicting the second-order inelastic response of the semi-rigid frame shown in Fig. 4.7, in which NASF is a nonlinear finite element program for second-order spread-of-plasticity analysis of semi-rigid planar steel frames (Nguyen, 2010). It can be seen that the nonlinear load – displacement curves agree well. The ultimate load factor obtained from the proposed analysis is 0.940 greater than 1.73% of 0.924 produced by NASF (Nguyen, 2010). Table 4.1 summarizes the comparison of the ultimate load factor of the frame with the rigid and semi-rigid connections using the proposed program and those results of previous studies. Less than 2% error was achieved.

Table 4.1 Comparison of ultimate load factor of Vogel portal frame

<table>
<thead>
<tr>
<th>Frame type Method</th>
<th>Ultimate load factor</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Plastic zone – Vogel (Vogel, 1985) (50ele column, 40ele beam)</td>
<td>1.022</td>
<td>–</td>
</tr>
<tr>
<td>Fiber beam-column – proposed (2 integration points)</td>
<td>1.009</td>
<td>-1.27</td>
</tr>
<tr>
<td>Semi-rigid Plastic zone – NASF (Nguyen, 2010) (50ele column, 40ele beam)</td>
<td>0.924</td>
<td>–</td>
</tr>
<tr>
<td>Fiber beam-column – proposed</td>
<td>0.940</td>
<td>+1.73</td>
</tr>
</tbody>
</table>
Fig. 4.6. Load – displacement curve of Vogel portal rigid frame with and without shear deformation

Fig. 4.7. Load – displacement curve of Vogel portal semi-rigid frame
Using the same personal computer configuration (AMD Phenom II X4 955 Processor, 3.2 GHz; 4.00 GB RAM), the analysis time of the proposed program and NASF for the second-order inelastic behavior of the semi-rigid frame are 37 sec and 114 sec, respectively. The analysis time of NASF is 3.08 times longer than the proposed program. This result demonstrates the higher computational efficiency of the proposed program.

4.1.2 Stelmack Experimental Two-Story Steel Frame

A one-bay, two-story semi-rigid steel frame was tested by Stelmack (Stelmack, 1982) shown in Fig. 4.8, and it is selected to verify the present study with experimental test results. The A36 W5x16 hot-rolled steel was used for all frame members. The three parameters of the Kishi-Chen power model are used to determine a curve-fitting as follows: $R_{ki} = 4,463 kN \cdot m / rad$, $M_u = 26 kN \cdot m$, and $n = 0.87$. The moment-rotation relationship of the connections by the experiment and curve-fitting data are in agreement as shown in Fig. 4.9. Gravity loads were first applied at the third points of
the beam of the first floor, and then a lateral load was applied as the second loading sequence. All members are discretized into sixty six fibers (twenty four at both flanges, eighteen at the web) on the cross section and two integration points at their two ends. The lateral load-displacement curves obtained by the proposed program and experimental work compare well in Fig. 4.10. As a result, the proposed analysis is adequate in predicting the behavior and strength of semi-rigid connections.

![Connection moment vs. rotation curve](image)

**Fig. 4.9.** Moment – rotation curve of connections by Stelmack experiment and curve fitting
4.1.3 **Vogel Six-Story Steel Frame**

This steel frame was also solved by Vogel (Vogel, 1985) as the European calibration frame for considering both the effects of nonlinear geometry and spread of plasticity. The configuration of the frame is illustrated in Fig. 4.11. The geometric imperfections are simulated by the initial out-of-plumb straightness of $\psi = 1/450$ for all column members and the initial ECCS residual stress. Young’s modulus and the yield stress of steel are $E = 20500N/mm^2$ and $\sigma_y = 235N/mm^2$. In this analysis, the proposed program uses five elements per beam member, one element per column member, and five integration points per element. The cross sections of all members are discretized into sixty six fibers (twenty four at both flanges, eighteen at the web). For the frame with rigid connections, the results of the load-displacement curve and the ultimate load factor are compared in Fig. 4.12 and Table 4.2, respectively. Less than 2% error was
achieved. The proposed fiber beam-column element result of 1.100 is slightly lower than the ultimate limit load of 1.110 by Vogel’s plastic zone method. It can be concluded that the proposed program accurately predicts the nonlinear behavior and strength of steel frames considering both the geometric and material nonlinearities.

![Fig. 4.11. Vogel six-story frame with semi-rigid connections](image)

Chui and Chan (Chan and Chui, 2000) built in the semi-rigid joints at beam ends to study a more realistic nonlinear behavior of the frame, as shown in Fig. 4.11. The curve fitted parameters of the Chen-Lui exponential model for both flush end plate and single web angle connections were listed in Table 4.3. Fig. 4.13 shows the nonlinear moment-
rotation relationship of these connections. The results of the proposed analysis are similar with those of Chan and Chui, as illustrated in Fig. 4.14. It can be seen that the proposed analysis predicts the load-displacement curves are slightly lower than those of Chan and Chui by the refined plastic hinge method. This difference is due to the fact that Chan and Chui’s method employed approximate shape functions accounting for the effects of geometric nonlinearity, while the proposed method uses the stability functions for capturing the second-order effects accurately. Moreover, the proposed analysis can monitor spread of plasticity along the members through selected integration points and the initial residual stress distribution is adequately considered.

![Load – Displacement Curve](image)

**Fig. 4.12.** Load – displacement curve of Vogel six-story rigid frame

(PZ – Plastic Zone, RPH – Refined Plastic Hinge)
Table 4.2 Comparison of ultimate load factor of Vogel six-story rigid frame

<table>
<thead>
<tr>
<th>Method</th>
<th>Ultimate load factor</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic zone – (Vogel, 1985)</td>
<td>1.110</td>
<td>–</td>
</tr>
<tr>
<td>Plastic hinge – (Vogel, 1985)</td>
<td>1.120</td>
<td>+0.90</td>
</tr>
<tr>
<td>Refined plastic hinge – (Chan and Chui, 2000)</td>
<td>1.125</td>
<td>+1.35</td>
</tr>
<tr>
<td>Fiber beam-column – proposed</td>
<td>1.100</td>
<td>-0.90</td>
</tr>
</tbody>
</table>

Table 4.3 Parameters of connections for the Chen-Lui exponential model

<table>
<thead>
<tr>
<th>Semi-rigid connections</th>
<th>A - Single web angle</th>
<th>C - Flush end plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested by</td>
<td>(Chen and Lui, 1987)</td>
<td>(Ostrander, 1970)</td>
</tr>
<tr>
<td>M0 (kN.m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Rkf (kN.m/rad)</td>
<td>5.322</td>
<td>108.925</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.00051167</td>
<td>0.00031783</td>
</tr>
<tr>
<td>C1</td>
<td>-4.892</td>
<td>-28.287</td>
</tr>
<tr>
<td>C2</td>
<td>137.140</td>
<td>573.189</td>
</tr>
<tr>
<td>C3</td>
<td>-661.841</td>
<td>-3433.984</td>
</tr>
<tr>
<td>C4</td>
<td>1465.397</td>
<td>8511.301</td>
</tr>
<tr>
<td>C5</td>
<td>-1510.926</td>
<td>-9362.567</td>
</tr>
<tr>
<td>C6</td>
<td>590.000</td>
<td>3832.899</td>
</tr>
<tr>
<td>Rki (kN.m/rad)</td>
<td>5440.592</td>
<td>12340.198</td>
</tr>
</tbody>
</table>
Fig. 4.13. Moment-rotation curve of semi-rigid connections for Vogel six-story frame

Fig. 4.14. Load – displacement curve of Vogel six-story frame with different beam-to-column connections

with different beam-to-column connections
4.1.4 Orbison Six-Story Space Steel Frame – A Case Study

(a) Plan view

(b) Perspective view

Fig. 4.15. Orbison six-story space frame with semi-rigid connections
A six-story space rigid frame was firstly analyzed by Orbison et al. (Orbison, McGuire and Abel, 1982) using a plastic hinge approach. Recently, Chiorean (Chiorean, 2009) used a beam-column method considering the spread of plasticity along the member length and the effects of connection flexibility. The geometric properties are described in Fig. 4.15. A36 steel with the yield stress of 250 MPa, Young’s modulus of 206,850 MPa, and a shear modulus of 79,293 MPa are used for all members. The bolted top and seat angle connections were assumed for all beam-to-column connections of the frame and their parameters using Kishi-Chen power model are listed in Table 4.4 (Chiorean, 2009). Both the weak-axis and strong-axis beam-to-column connections are considered in this study. A uniform floor load of 9.6 kN/m² is converted into equivalent concentrated loads on the top of the columns. Wind loads are simulated by point loads of 53.376 kN in the Y-direction at every beam-column joints. One proposed beam-column element with five integration points per member are used to model this structure. The cross sections of all members are discretized into sixty six fibers (twenty four at both flanges, eighteen at the web). As shown in Fig. 4.16, the load – displacement curve at roof node A predicted by the proposed program compare well with Chiorean’s result.

To consider initial member out-of-straightness by using only one element per member, further-reduced tangent modulus method presented by (Kim and Chen, 1996), (Chen and Kim, 1997) can be employed. In the proposed program, Young’s modulus of 0.85*E is directly assigned for all steel members to consider initial member out-of-straightness. Fig. 4.17 shows the load-displacement curves of the frames with and without geometry imperfection. The ultimate load factors of the nonlinear semi-rigid frame with and without initial member out-of-straightness are 0.857 and 0.841,
respectively. The ultimate load of the frame reduces approximate to 1.97% as considering initial member out-of-straightness. It can be concluded that the proposed program is reliable in predicting the nonlinear inelastic behavior of 3D semi-rigid steel frames.

![Fig. 4.16. Load – displacement curve at Y-direction node A of Orbison 3-D six-story frames](image)

Table 4.4 Parameters of semi-rigid connections follow the Kishi-Chen model

<table>
<thead>
<tr>
<th>Beam section</th>
<th>Bending-axis</th>
<th>$M_u$ (kN.m)</th>
<th>$R_{si}$ (kN.m/rad)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12x87</td>
<td>Strong-axis</td>
<td>300</td>
<td>160,503.2</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>300</td>
<td>52,267.75</td>
<td>1.57</td>
</tr>
<tr>
<td>W12x53</td>
<td>Strong-axis</td>
<td>300</td>
<td>92,185.09</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>300</td>
<td>20,776.82</td>
<td>1.57</td>
</tr>
<tr>
<td>W12x26</td>
<td>Strong-axis</td>
<td>200</td>
<td>44,247.8</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>Weak-axis</td>
<td>200</td>
<td>3,752.54</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Fig. 4.17. Load – displacement curve at Y-direction node A of Orbison 3-D six-story frames with and without initial member out-of-straightness

4.2 Dynamic Problems

A structural analysis program written in FORTRAN programming language is developed based on the above-mentioned formulations to predict nonlinear inelastic time-history responses of space semi-rigid steel frames subjected to dynamic loadings. Earthquake records shown in Fig. 2.8 are used as ground excitation in dynamic analysis. Their peak ground accelerations and time steps are listed in Table 2.1. It is verified for accuracy and reliability by the comparison of predicted results with those generated by ABAQUS (DSSC, 2010), SAP2000 (CSI, 2011), and the Stelmack experimental test (Stelmack et al., 1986) of a two-story steel frame. The elastic-perfectly plastic steel material is used in this study. Each member is modeled by one element which is integrated along the member length using the Gauss-Lobatto quadrature rule. All member cross-sections are divided into 66 fibers (48 fibers at each flange, 18 fibers at
the web). In the dynamic analysis using the HHT method, the coefficient $\alpha = 0$ is adopted.

4.2.1 Portal Steel Frame subjected to Earthquakes

![Diagram of Portal Frame](image)

Considering a portal frame with masses lumped at the frame nodes subjected to the Loma Prieta and San Fernando earthquakes, the geometry and material properties of the frame are illustrated in Fig. 4.18. Earthquake records are shown in Fig. 2.8 and Table 2.1. In the numerical modeling of the proposed program, each frame member is modeled by one element with two integration points at the member ends. Meanwhile, the ABAQUS program uses 40 discrete elements per each B22 Timoshenko beam member since the ABAQUS analysis can not accurately capture the second-order inelastic response of the frame if only a few elements per member are used in the modeling. All elements are divided into 66 fibers (48 fibers at each flange, 18 fibers at the web) on the cross section in the proposed program.
After performing the vibration analysis, the first two natural periods along the applied earthquake direction of the portal frame are obtained and compared in Table 4.5. It can be seen that a strong agreement of natural periods of the frame generated by ABAQUS and the proposed program is obtained. These two natural periods are used to determine the Rayleigh damping coefficients (shown in Table 4.5) by assuming the equivalent viscous damping ratio $\xi$ of 5% in the next time-history analysis step.

Table 4.5 Comparison of periods and Rayleigh damping coefficients of portal frame

<table>
<thead>
<tr>
<th>Program</th>
<th>1st period (sec)</th>
<th>2nd period (sec)</th>
<th>$\xi$</th>
<th>$\alpha_M$</th>
<th>$\beta_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABAQUS</td>
<td>0.81619</td>
<td>0.029069</td>
<td>0.05</td>
<td>0.74334</td>
<td>446.7*10^-6</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.82066</td>
<td>0.028954</td>
<td>0.05</td>
<td>0.73953</td>
<td>445.1*10^-6</td>
</tr>
<tr>
<td>Diff. (%)</td>
<td>0.55</td>
<td>-0.40</td>
<td>0.00</td>
<td>-0.51</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

Without considering the initial residual stress, the second-order elastic and second-order inelastic time-displacement responses of the frame under the two earthquakes of Loma Prieta and San Fernando are compared in Fig. 4.19 and Fig. 4.20, respectively. A comparison of the peak displacements is given in Table 4.6 with the maximum difference of 1.55%. It can be observed that the proposed program and ABAQUS generate nearly identical results in all cases, including the permanent drifts of displacement due to the gradual yielding behavior in the second-order inelastic analysis cases. The discrepancy in displacement response of the second-order elastic (SE) and second-order inelastic (SI) analyses is relatively clear.

In considering the effect of the initial ECCS residual stress (Kim, Ngo-Huu and Lee, 2006), Fig. 4.21 shows the comparison of the second-order inelastic responses of the frame. It can be seen that the permanent drifts of the roof floor have significant
differences in the two cases. It can be concluded that the effect of initial residual stresses should be considered in advanced analysis.

Using the same personal computer configuration (AMD Phenom II X4 955 Processor, 3.2 GHz; 8.00 GB RAM), the computational time of the proposed program and ABAQUS for the second-order inelastic responses of the frame subjected to the San Fernando earthquake are 1 min 37 sec and 24 min 48 sec, respectively. The computational time of ABAQUS is about 15 times longer than the proposed program. This result demonstrates the computational efficiency of the proposed program.

Table 4.6 Comparison of peak displacements (mm) of portal frame

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Max/Min</th>
<th>Analysis type</th>
<th>ABAQUS</th>
<th>Proposed</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loma Prieta</td>
<td>Max</td>
<td>Elastic</td>
<td>104.41</td>
<td>104.56</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>104.71</td>
<td>103.35</td>
<td>-1.30</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-88.15</td>
<td>-87.90</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-84.42</td>
<td>-84.78</td>
<td>0.43</td>
</tr>
<tr>
<td>San Fernando</td>
<td>Max</td>
<td>Elastic</td>
<td>119.47</td>
<td>120.32</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>122.44</td>
<td>124.34</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Elastic</td>
<td>-93.44</td>
<td>-94.51</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inelastic</td>
<td>-79.04</td>
<td>-79.26</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Fig. 4.19. Displacement time-history responses of portal frame under Loma Prieta earthquake

(a) Second-order elastic analysis

(b) Second-order inelastic analysis
Fig. 4.20. Displacement time-history responses of portal frame under San Fernando earthquake

(a) Second-order elastic analysis

(b) Second-order inelastic analysis

Fig. 4.20. Displacement time-history responses of portal frame under San Fernando earthquake
Fig. 4.21. Second-order inelastic responses of portal frame with and without residual stress

(a) Loma Prieta

(b) San Fernando
4.2.2 Experimental Two-Story Steel Frame subjected to Cyclic Loadings

(a) Configuration

(b) Lateral load history
This numerical example is to verify the proposed connection modeling with the results of the experimental test and numerical analysis of Stelmack et al. (1986) (Stelmack, Marley and Gerstle, 1986). The geometry and applied loads of the test frame are shown in Fig. 4.22a and Fig. 4.22b. All frame members are W5×16 A36 steel. The ECCS residual stress distribution (Kim, Ngo-Huu and Lee, 2006) is assigned for steel cross-sections, assuming that residual stresses uniformly distribute along each fiber length. Young’s modulus of the material is 29,000 ksi. 1/2-in. angle connections were used in the test, and 14 monotonic moment-rotation test results of such connections were reported by Stelmack et al. (Stelmack, Marley and Gerstle, 1986). The upper and lower bounds obtained from the test data are shown in Fig. 4.22c. The trilinear model was utilized by Stelmack et al. while the Richard-Abbott model is used in the present study, as plotted in Fig. 4.22c. In the Stelmack study, kinematic hardening model
represent cyclic behavior of connections while independent hardening model is employed in this study. The four parameters of the Richard-Abbott model are determined by a curve-fitting procedure as follows: \( R_{ki} = 19,500 \text{kip} \cdot \text{in} / \text{rad} \), \( R_{kp} = 750 \text{kip} \cdot \text{in} / \text{rad} \), \( M_o = 150 \text{kip} \cdot \text{in} \), and \( n = 1.56 \). The lateral load cycles begin with loads applied to the first story of \( \pm 1 \) kip and increased by 1 kip increments up to \( \pm 5 \) kips as shown in Fig. 4.22b, the second-story load is a half of the first-story load; and, the total load procedure is divided into 500 steps.

The experimental and analytical results of the moment-rotation loops at the connection C are separately compared for each load cycle as shown in Fig. 4.23. It can be seen that the results predicted by the proposed analysis using the Richard-Abbott model are relatively close to the experimental results and smoother than the results of Stelmack et al. using the trilinear model. It may be concluded that the proposed program can generate acceptably accurate and smooth moment-rotation curves in predicting the nonlinear behavior of steel frames with semi-rigid connections.
Fig. 4.23. Comparison of moment-rotation curves at connection C of Stelmack experimental frame

4.2.3 Space Six-Story Steel Frame – A Case Study

A six-story space steel frame with semi-rigid beam-to-column connections is shown in Fig. 4.15. Nonlinear inelastic static analysis was performed by Chiorean (Chiorean, 2009), Ngo-Huu et al. (Ngo-Huu, Nguyen and Kim, 2012), and Nguyen and Kim (Nguyen and Kim, 2014). The nonlinear elastic dynamic behavior was investigated by Nguyen and Kim (Nguyen and Kim, 2013). This study investigates nonlinear inelastic time-history responses of the frame subjected to the El Centro and San Fernando earthquakes, their records are shown in Fig. 2.8 and Table 2.1. The elastic modulus, shear modulus, and yield stress of steel are 206,850 MPa, 79,293 MPa, and 250 MPa, respectively. Three parameters of the Kishi-Chen power model for the semi-rigid connections calculated by Chiorean (Chiorean, 2009) are listed in Table 4.4. The lumped masses of 128.42 and 256.84 kN.sec²/m are transferred from the uniform floor.
pressure of 9.6 kN/m², and they are assigned at the frame nodes as shown in Fig. 4.15b. In the proposed program, each frame member is modeled by one element with five integration points along the member length. The earthquake excitations are applied in the direction Y. Structural viscous damping is considered by using the Rayleigh damping matrix with a damping ratio $\xi$ of 0.05.

Fig. 4.24 and Fig. 4.25 show the lateral displacement responses at the node A/appropriating to the linear and nonlinear semi-rigid frames under the El Centro and San Fernando earthquakes, respectively. Results of the peak displacements are compared in Table 4.7. It can be observed that a strong agreement of displacements predicted by the proposed program and SAP2000 is obtained in the second-order elastic analysis. The proposed program uses the spread-of-plasticity approach considering directly the effect of initial ECCS residual stress (Kim, Ngo-Huu and Lee, 2006), whereas the SAP2000 program (CSI, 2011) uses the lumped-plasticity approach. Nonlinear inelastic displacement responses of the proposed program and SAP2000 in the second-order inelastic analysis have various permanent shifts of displacements. The proposed program is more accurate than SAP2000 because it can capture the real behavior of structures by taking into account for the effects of spread-of-plasticity, initial residual stresses, and nonlinear connections.

The permanent displacement of the nonlinear semi-rigid frame is less than that of the linear semi-rigid frames because of the effect of hysteretic damping of the nonlinear semi-rigid connections. It can be concluded that the beam-to-column connections play a significant role in the earthquake resistant design. Predicting accurately the real behavior of framed structures leads to both safer and more economic designs.
In this case study, the combined effect of inelastic hysteretic damping due to spread of plasticity, hysteresis loops of semi-rigid connections, residual stress, and initial member geometric imperfections acting on overall structural responses is investigated. The ECCS initial residual stress is assumed (Kim, Ngo-Huu and Lee, 2006). Initial member out-of-straightness is considered by employing the reduced tangent modulus method proposed by (Kim and Chen, 1996) and (Chen and Kim, 1997). In the proposed program, Young’s modulus of $0.85 \times E$ is directly assigned for all steel members to consider initial member out-of-straightness. The frame with nonlinear semi-rigid connections subjected to the two earthquakes is analyzed for four cases (case 1 – without residual stress and initial member out-of-straightness, case 2 – considering only residual stress, case 3 – considering only initial member out-of-straightness, and case 4 – considering both residual stress and initial member out-of-straightness). As shown in Fig. 4.26, there are no significant differences in the second-order inelastic displacement responses between the frame models that include the initial residual stress and those without this effect, whereas the initial member out-of-straightness strongly acts on the final behavior of the frame. It can be concluded that the effect of initial member out-of-straightness is more important than the effect of residual stress. Fig. 4.27 plots nonlinear moment-rotation responses at the connection C corresponding to the four cases. It can be seen that the hysteresis loops are different because member-force redistribution caused by gradual yielding of framed members is different. It can be concluded that the proposed program is reliable in predicting the nonlinear inelastic behavior of space semi-rigid steel frames.
(a) Second-order elastic responses of the semi-rigid frame

(b) Second-order inelastic responses of the semi-rigid frame

Fig. 4.24. Nonlinear time-history responses of six-story space steel frame subjected to El Centro earthquake
(a) Second-order elastic responses of the semi-rigid frame

(b) Second-order inelastic responses of the semi-rigid frame

Fig. 4.25. Nonlinear time-history responses of six-story space steel frame subjected to San Fernando earthquake
Table 4.7 Comparison of peak displacements (mm) at node A of six-story space steel frame

<table>
<thead>
<tr>
<th>Earthquakes</th>
<th>Max/min</th>
<th>Frame type - Analysis type</th>
<th>Displacement (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SAP2000</td>
<td>Present</td>
<td></td>
</tr>
<tr>
<td>El Centro</td>
<td>Max</td>
<td>LC - NE</td>
<td>338.22</td>
<td>330.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>238.07</td>
<td>238.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>245.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>242.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>LC - NE</td>
<td>-442.89</td>
<td>-441.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>-419.30</td>
<td>-390.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>-326.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>-343.74</td>
<td></td>
</tr>
<tr>
<td>San Fernando</td>
<td>Max</td>
<td>LC - NE</td>
<td>152.69</td>
<td>151.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>73.40</td>
<td>83.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>92.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>90.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>LC - NE</td>
<td>-125.38</td>
<td>-124.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LC - NI</td>
<td>-203.02</td>
<td>-174.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NE</td>
<td>-72.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC - NI</td>
<td>94.16</td>
<td></td>
</tr>
</tbody>
</table>

Note: Rigid Connections (RC), Linear semi-rigid Connections (LC), Nonlinear semi-rigid Connections (NC); Nonlinear Elastic analysis (NE), Nonlinear Inelastic analysis (NI)
Fig. 4.26. Second-order inelastic displacement responses of six-story space steel frame under two earthquakes considering geometric imperfections.
Fig. 4.27. Moment-rotation responses at connection C of six-story space steel frame under two earthquakes considering geometric imperfections.
5. Summary and Conclusions

An accurate and effective numerical procedure for the nonlinear inelastic static and time-history analysis of semi-rigid steel frames accounting for three main sources of both nonlinearities and damping is successfully developed. The geometric nonlinearities are considered by using the stability functions and the geometric stiffness matrix. The spread-of-plasticity behavior is captured by monitoring the uniaxial stress-strain relation of each fiber on the cross section and using only two to five Gauss-Lobatto integration points along the member length. An independent zero-length connection element comprising six translational and rotational springs, in order to connect six degrees of freedom of two identical nodes, is developed to simulate the semi-rigid beam-to-column connections. The independent hardening model is employed for the cyclic behavior of semi-rigid connections. Three major sources of damping are considered: structural viscous damping, hysteretic damping due to gradual yielding of material, and hysteretic damping due to hysteresis loops of nonlinear connections. The accuracy and efficiency of the proposed procedure are proved by comparing the results with the commercial finite element packages ABAQUS and SAP2000, and the Stelmack experimental test (Stelmack, Marley and Gerstle, 1986). The following conclusions can be drawn from the present study:

- By using only one element per member and two to five Gauss-Lobatto integration points along the member length, the proposed method can accurately capture the second-order effects and spread-of-plasticity, save computer resources, and reduce computational time.
- The proposed connection element can be developed to simulate various structure
connections (e.g., truss joints, bridge joints, concrete joints, damping, etc.). The proposed connection element can also be used to simulate the steel column base connections.

- The flexibility of nonlinear semi-rigid connections plays a major role in predicting the real behavior of steel frames under dynamic loadings.

- Initial geometric imperfections and residual stresses are important factors needed to be considered in the practical design of steel frames.

- The proposed program can be used in lieu of sophisticated finite element software packages as ABAQUS and ANSYS.

- In addition, the proposed program needs to be improved for considering the effects of panel-zone deformation, warping torsion, local buckling, lateral-torsional buckling, etc. with aims of developing a performance-based practical design program of large-scale 3D steel structures subjected to extreme loadings.
Chapter 5. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

1. Summary and Conclusions

In this dissertation, three advanced analysis approaches are proposed for predicting the second-order inelastic behavior of semi-rigid steel framed structures subjected to static and dynamic loadings. Three major sources of nonlinearity are considered in the analyses as follows: (1) material nonlinearity; (2) geometric nonlinearity; and (3) connection nonlinearity. Three main sources of damping are also considered in the analyses as follows: (1) hysteretic damping due to inelastic materials; (2) structural viscous damping (Rayleigh damping); and (3) hysteretic damping due to hysteresis loops of nonlinear connections. Three types of analysis provided in the proposed programs are: (1) nonlinear elastic and inelastic static analysis including rigid, linear semi-rigid, or nonlinear semi-rigid connections; (2) nonlinear elastic and inelastic time-history analysis including rigid, linear semi-rigid, or nonlinear semi-rigid connections; and (3) free vibration analysis. The proposed programs can be used to assess realistically both strength and behavior of steel framed structures and their component members in a direct manner.

In the Chapter II, a nonlinear inelastic beam-column finite element formulation including semi-rigid connections is developed for nonlinear time-history analysis of planar steel frames. The Hermite interpolation functions are utilized to find out displacements along the member length. The elemental tangent stiffness matrix is modified for considering the flexibility of semi-rigid connections. Several sub-elements
per member are used to take into account for distributed plasticity and the second-order
effects. The Newmark numerical integration method combined with the Newton-
Raphson equilibrium iterative method is employed for solving nonlinear equations of
motion.

In the Chapter III, a nonlinear inelastic beam-column finite element formulation
based on plastic hinges and stability functions is presented for nonlinear inelastic static
and dynamic analysis of space semi-rigid steel frames. The geometric nonlinearity
cau sed by the interaction between axial force and bending moment is taken into account
using the stability functions, while the material nonlinearity is captured using the
refined plastic hinge model. The benefit of employing the stability functions and the
refined plastic hinge model is that it can accurately capture the nonlinear effects by
modeling one or two element per member, and hence this leads to a high computational
efficiency compared to the conventional finite element method using the interpolation
functions. The spread-of-plasticity and residual stresses are indirectly considered using
the reduced tangent modulus.

In the Chapter IV, a nonlinear inelastic beam-column finite element formulation
using stability functions is presented for nonlinear inelastic static and dynamic analysis
of space semi-rigid steel frames. The spread-of-plasticity is captured by dividing into
several fibers on the cross sections and monitoring throughout the member length using
selected integration points. Residual stresses can be directly assigned for fibers as initial
stresses. The benefit of employing the stability functions is that using only one element
per member can accurately capture the second-order effects.
A space multi-spring element is developed to simulate semi-rigid beam-to-column connections. The mathematic models proposed from curve fitting of static experimental tests of beam-to-column joints are employed for skeleton curves of the moment-rotation relationship of semi-rigid connections. The cyclic behavior of connections is traced by the independent hardening model.

For the nonlinear static analysis, the generalized displacement control method is used to solve nonlinear equilibrium equations because of its general numerical stability and efficiency. This algorithm can accurately trace the equilibrium path of nonlinear problems with multiple limit points. For the nonlinear time-history analysis, the Hilbert-Hill-Taylor method combined with the Newton-Raphson equilibrium iterative method is adopted for solving nonlinear equations of motion. The accuracy and computational efficiency of the proposed programs are verified throughout a wide range of numerical examples by comparing the obtained results with those predicted by ABAQUS, SAP2000, and other results available in the literature.

Based on the results of this study, the conclusions can be made as follows:

The nonlinear inelastic 2-D beam-column finite element including semi-rigid connections is developed for nonlinear time-history analysis of planar steel frames in the Chapter II. The effects of distributed plasticity, geometric nonlinearities, flexibility of connections, bowing are directly considered in the elemental tangent stiffness matrix. These effects can be captured accurately by using several sub-elements per member. A structural analysis program named NSAP (Nonlinear Structural Analysis Program) was successfully developed using the procedure mentioned in the Chapter II.
The Chapter III and IV presented the innovations for the Practical Advanced Analysis Program (PAAP) in detail. The PAAP program can accurately and computationally efficiently perform the nonlinear inelastic behavior of three-dimensional steel frames with semi-rigid connections subjected to static and dynamic loadings. The second-order spread-of-plasticity beam-column element and the beam-to-column connection element are successfully added to the element library of PAAP program. Using only one element per member with two to five integration points along the member length, it can be capture exactly the spread-of-plasticity behavior of steel framed structures. Thus, it can be concluded that the PAAP program promises to be a valuable tool not only for research but also for daily use in practical design.

2. Recommendations

Based on the results of this study, some recommendations are suggested for future works as follows:

Most current analysis programs using the beam-column methods are unable to model the inelastic lateral-torsional buckling and warping effects directly. For this reason, the slenderness ratios of all members must be checked depending on the suggestions in the specification. Therefore, the future research should properly consider these effects directly in the formulation of beam-column element.

The effects of panel zone and axial shortening due to member curvature bending (bowing effects) are ignored in this study. The panel zone is the position which connects column to column of two different stories and column to beam. In practical analysis and design, the panel zone is usually assumed to be perfectly rigid for simplify. However, the flexibility of panel zone is finite. The future work should include these effects.
In this study, the proposed beam-to-column element can be used to modeling complicated space connections such as truss joints, bridge joints, concrete joints, etc. However, it only is used for simulating stiffness of nonlinear rotational springs.

Since the application of the proposed program is limited to the structural members made from steel material, the structural members made from concrete material is not considered herein. Thus, the high priority should be given to the development of material library as well as element library of the proposed program so that it can be applied to a more wide range of structures. So the steel-concrete composite beam-column element can be developed in the future study.

In recent decades, the structural members with concrete-filled steel tube (CFT) section have become popular due to their excellent performances such as high ductility and improved strength without increasing the member size. Several computational models have been developed to represent the behavior of CFT member. Among them, a flexibility-based fiber model is the most promising one for the second-order inelastic analysis of CFT member. The proposed plastic-fiber beam-column element can be upgraded for the CFT member.

In the nonlinear time-history analysis, it has been acknowledged that Rayleigh damping lack physical consistency, hence, it must be carefully used to avoid encountering unintended consequences as the appearance of artificial damping. Since the stiffness matrix of structures is change step-by-step, which type of structural stiffness matrix should be used to calculate Rayleigh damping for analyzing nonlinear structures? (e.g. the initial stiffness matrix, the tangent stiffness matrix, etc.), it is
necessary to investigate this phenomenon. The consequence of Rayleigh damping in the inelastic analysis of structural systems should be investigated carefully.
REFERENCES

Alemdar, B. N. & White, D. W. (2005), Displacement, Flexibility, and Mixed Beam–
Column Finite Element Formulations for Distributed Plasticity Analysis, Journal of
Structural Engineering, 131(12), 1811-1819.

Flexible Beam-Column Connections, Canadian. Journal of Civil Engineers, 11, 245-
254.

Awkar, J. C. & Lui, E. M. (1999), Seismic Analysis and Response of Multistory
Semirigid Frames, Engineering Structures, 21(5), 425-441.

Azizinamini, A. & Radziminski, J. B. (1989), Static and Cyclic Performance of
Semirigid Steel Beam-to-Column Connections, Journal of Structural Engineering-
Asce, 115(12), 2979-2999.

Problems, International Journal for Numerical Methods in Engineering, 14(8), 1262-
1267.

Bergan, P. G. (1980), Solution Algorithms for Nonlinear Structural Problems,
Computers & Structures, 12(4), 497-509.

Techniques for Non–Linear Finite Element Problems, International Journal for
Numerical Methods in Engineering, 12(11), 1677-1696.

Chan, S. L. & Chui, P. P. T. (2000), Nonlinear Static and Cyclic Analysis of Steel
Frames with Semi-Rigid Connections, Elsevier.

Chen, W. F. & Kim, S. E. (1997), Lrfd Steel Design Using Advanced Analysis, CRC

Chen, W. F. & Kishi, N. (1989), Semirigid Steel Beam-to-Column Connections - Data-


Elsevier, New York.


CSI. (2011), Sap2000, Linear and Nonlinear Static and Dynamic Analysis and Design of Three-Dimensional Structures, Berkeley, California, USA, Computers and Structures, Inc.

DSSC. (2010), Abaqus/Cae 6.10-1, Dassault Systèmes Simulia Corporation, USA.


Nguyen, P. C. (2010), Nonlinear Analysis of Planar Semi-Rigid Steel Frames Subjected to Earthquake Excitation by Plastic-Zone Method, Faculty of Civil Engineering, Faculty of Civil Engineering, Ho Chi Minh City University of Technology, Vietnam.


Ostrander, J. R. (1970), An Experimental Investigation of End-Plate Connections, University of Saskatchewan, Saskatoon, Saskatchewan, Canada.


Popov, E. P. (1983), Seismic Moment Connections for Moment-Resisting Steel Frames, Report No. UCB/EERC-83/02, Earthquake Engineering Research Centre, University of California, Berkeley, CA.


Thai, H. T. & Kim, S. E. (2009), Practical Advanced Analysis Software for Nonlinear Inelastic Analysis of Space Steel Structures, Advances in Engineering Software, 40(9), 786-797.


국문초록

정적하중 및 동적하중을 받는 3차원 반갑접 강뼈대
구조물의 고등해석

Nguyen Phu Cuong
건설환경공학과
세종대학교 대학원

이 논문은 극한강도와 정적 및 동적 하중을 받는 비선형 빔-기둥 결합에 강뼈대
구조물의 기동을 정확하고 효율적으로 포착할 수 있는 세가지 다양한
고등해석방법을 제시한다. 이 프로그램에 적용된 3 가지 비선형 요소는: (1)재료
비선형; (2)기하학적 비선형; 그리고 (3)접합 비선형 요소이다. 3 종류의 재료 및
기하학적 비선형을 고려한 비선형 보-기둥 요소는 두 비선형 구조 분석
프로그램으로 구분된다: (1) 비선형 구조 해석 프로그램 (NSAP) - 2-D 소성영역
유한 요소; (2) 실제 고급 분석 프로그램 (PAAP) - 3-D 개선소성현지요소와 3-D
소성심유소. 제안된 프로그램에 의해 분석한 강뼈대 구조는 3 가지 종류가
있다: (1) 강접 프레임-빔-기둥 연결들은 완전히 강성; (2) 선형 반강접 프레임-빔-
기둥 연결은 일정한 강성을 갖는다. 그리고 (3) 비선형 반강접 프레임-빔-기둥
연결은 연속적으로 변할 수 있는 강성이다. 세가지 종류의 해석이 수행될 수 있다.
(1) 비선형비탄성 정적해석; (2) 비선형탄성 및 비탄성 시간 이력 해석; 그리고 (3)
자유 진동 분석. 감쇠의 세가지 주요 요소는 제안된 프로그램에 반영된다: (1) 비선형 재료의 시간이력 감쇠; (2) 레일리 감쇠를 이용한 구조적 점성감쇠; (3) 비선형 보-기둥 연결에 의한 시간이력 감쇠.

첫 번째 방법- 제안된 프로그램 NSAP을 사용하여 동적 및 지진 하중을 받는 보-기둥 접합부에서 평면의 강력 대 구조물의 비선형비탄성해석을 활용한 변위기반의 유한요소기법이 개발되었다. 기하학적 비선형효과, 굴착효과, 재료의 점진적인 항복, 그리고 비선형 접합의 유연성을 직접적으로 고려한 부분적인 변형경로, 탄소성 보-기둥 요소가 제안되었다. 선형 및 Hermite 보간 함수를 사용하여 포착된 기하학적 비선형성의 부재는 여러 하위 요소로 구분된다. 소성의 확산은 하위 요소 단면의 각 심유의 단축 응력-변형을 관계를 통하여 결정된다. 비선형 접합은 비선형 보-기둥 요소의 접선 강성행렬을 수정하여 무질이 회전 스프링에 의해 시뮬레이션 된다. Newmark 수치적분방법 및 Newton-Raphson 평형 반복 알고리즘의 조합에 따라 수치해석 방법은 운동의 미분방정식을 해결하기 위해 제안된다. 제안된 프로그램에 의해 예측된 비선형 동적 기동은 상용 유한요소 소프트웨어 ABAQUS 와 이전의 연구와 비교하였다.

두 번째 및 세 번째 접근방법- 재료 비선형 개선 소성현지 모델 (개선 소성현지요소-RPH)나 가소성모델(소성섬유요소-PF)의 확산이 포착되는 동안 실질적인 고급 분석을 위해 제안된 프로그램 PAAP을 사용하여, 축력 및 굴림 모멘트는 사이의 변호 작용에 기인한 기하학적 비선형성은 안정성 함수에 의하여 고려되어 진다. 안정함수와 개선 소성현지모델의 이점을 하나 또는 두개의...
요소를 모델링함으로써 비선형성의 효과를 포착할 수 있고, 형상함수를 사용한
유한요소법에 비해 높은 계산 효율을 갖는다. 일부의 경우, 높은 정밀도의 래벨을
구하는 것이 필요하고, 또한 점진적 소성화 모델을 사용해야 부재의 더 많은
정보를 줄 수 있다. 정확하게 점진적 소성화 효과를 관찰하려면, 부재는 부재의
길이와 단면의 심유를 따라 통합지점을 통하여 관찰된다. 반강점 연결의 비선형
기동을 고려하기 위해서 세가지 변환 스프링과 세가지 회전 스프링 공간의 연결
요소가 보에 보-기동 연결부위를 시뮬레이션 하기 위해 개발되고 있다.

이 논문에서는 비선형적평형방정식을 풀기 위하여 일반적인 수치 안정성과
효율성 때문에 수정된 Newton-Raphson 방법 및 일반화된 변위제어방법이
사용된다. 일반화된 변위제어 방법은 여러 한계점과 회복점을 비선형 현상의
평형경로를 정확하게 추적 할 수 있다. 수정된 Newton-Raphson 방법은 동적
해석과정이 실행되지 전에 완전히 정적 하중을 부가하기 위하여 사용된다.
점진적 반복적인 해법 알고리즘은 Hilber-Hughes-Taylor 방법이나 Newton-
Raphson 평형방정식인 Newmark 직접 적분법은 운동방정식을 해결하기 위해
사용된다.

두 컴퓨터 프로그램이 개발되었다: (1) 비선형 구조 해석 프로그램 (NSAP) -
C++ 프로그래밍 언어로 작성된; (2) 실제적인 고급 분석 프로그램 (PAAP) -
FORTRAN 77 프로그래밍 언어로 작성된. ABAQUS 및 SAP2000 의 상용 유한
요소 해석 패키지 및 문헌에서 사용할 수 있는 다른 결과에 의해 생성 된 것과
예측 결과를 비교하여 정확성과 계산의 효율성을 위해 확인된다. 몇 가지 수치
예제를 통해, 비용과 시간이 많이 소요되는 상용 소프트웨어를 사용하는 대신에 제안된 프로그램 (PAAP)는 매일 연습 설계를 위한 안정적이고 효율적인 도구로 증명되었다.
YOUR KNOWLEDGE HAS VALUE

- We will publish your bachelor's and master's thesis, essays and papers
- Your own eBook and book - sold worldwide in all relevant shops
- Earn money with each sale

Upload your text at www.GRIN.com and publish for free